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INTERSTELLAR SPACE.

(George Darwin Lecture, delivered by Professor Charles Fabry, Assoc.R.A.S., on 1938 May 13.)

The question which we have to consider may be put in the following way: What is in between the stars? Over and above the purely geometrical idea of *space*, is it necessary to suppose that there is *something* in this space?

If this question had been propounded at various times in the past, the answers would have been very diverse. After Newton and the founders of Celestial Mechanics the answer would have been "Nothing," except perhaps an extremely tenuous medium, able to exercise a very slight resistance to the heavenly bodies in motion, like the resistance of the air to projectiles. When such a body appeared to deviate slightly from the path calculated solely from Newtonian forces, the possibility of action by a resisting medium was invariably invoked; but this force was never manifested with any certainty.

A little later, towards 1840, the reply of all physicists to the same question would have been "Ether!" This is the epoch when Lamé wrote that the ether was as necessary to the physicists as the air we breathe. One of the most remarkable properties of this ether was the impossibility of detecting its presence. Nevertheless, again in 1893 one of our greatest mathematical astronomers, Tisserand, published a memoir entitled "Sur le calcul des perturbations des comètes et des planètes en tenant compte de la résistance de l'éther."

We no longer believe in an ether, elastic and possibly resistant. At the beginning of the present century the only reasonable reply to our question would once again have been "Nothing." The close study of the perturbations in the movements of planets and comets appeared the only

means of detecting anything, and, further, even this hope could only be entertained within the limits of the solar system. The contents of interstellar space seemed definitely closed to our investigations.

But during the last thirty years the aspect of things has completely changed. No longer do we imagine a hypothetical medium between the stars. We detect the presence of things already known to us on the Earth as the result of direct experiment. The things which we study in physics and in chemistry can exist between the stars ; we have succeeded in detecting some of these elements, and we are presumptuous enough to say—"there is nothing there except what we know." Up till now this presumption has been justified.

Possible Elements in Interstellar Space.—The constituents of all matter are formed of electric particles and atoms. The following is a list of the objects which it is possible to find in *space* :—

1. Free electrons, positive or negative.
2. Atoms, neutral or ionised.
3. Molecules.
4. Agglomerations of molecules, forming corpuscles of various sizes, from particles far smaller than the wave-lengths of light up to grains of dust.
5. Celestial bodies, from the tiniest meteors up to the largest meteorites.

In addition, objects of a different nature are to be found :

6. Radiation (photons).
7. Cosmic Rays (the nature of which is still under discussion).

It is above all by the analysis of light that we may learn something about all these possible constituents of interstellar space. The light which we receive from a star has traversed a long path through space ; the various objects scattered about in space may affect the light, and the modifications thus produced enable us to discover what has acted on the light. The sum total of the observable effects may be classed as follows, noting which of the objects enumerated above can be detected for each class :—

A. Absorption lines in stellar spectra. These lines may detect certain constituents (not all) of classes 2 and 3 above (perhaps also 4 in very special cases).

B. General weakening (selective or otherwise) but without definite absorption lines, due either to true absorption or to a diffusion of the light. This effect is produced by all the elements, electric or material (1, 2, 3, 4, 5), but with different characteristics.

C. Observation of the reflected (diffused) light coming from one or more stars, or from all the stars, causing luminosity of certain regions of the sky, or of the whole sky (1, 2, 3, 4, 5).

D. Emission of a bright-line spectrum by excitation coming from a star (2, 3). This observation may also serve to disclose the existence of certain kinds of photons (6) in a specified region of space.

E. Collision of some object with the Earth. It may be possible to detect 1 (cathode rays), 5, 6, 7.

F. Forces exerted on celestial bodies—resistance to motion (1, 2, 3, 4, 5) or repulsive forces (6).

It is obvious that the subject is extremely vast. I shall have to confine myself to passing in review the various methods of investigation and to stating what each one has been able to reveal.

Interstellar Absorption Lines.—Light from all celestial bodies giving a continuous spectrum exhibits absorption lines; all celestial light has traversed regions containing absorbing matter in an uncondensed state (gas; independent atoms or molecules). In each case the question arises, *Where* has the absorption been produced?

The oldest and most familiar case is that of sunlight, which we can study with very high dispersions, the spectrum showing more than 20,000 lines. The problem of their origin has been solved as a result of a series of researches beginning a hundred years ago with the work of Brewster. The absorption lines are of two kinds, one kind produced in the Sun's atmosphere and the other in the Earth's atmosphere. As is well known, the Doppler effect gives us the most certain way of distinguishing the two.

In the case of the planets we have three kinds of lines—solar, terrestrial and planetary. It is these latter which are of interest for the physical study of the planets; they have been detected *by elimination*, regarding as planetary any line which is neither solar nor terrestrial. But there is one difficult case: it is that of lines which may be both planetary and terrestrial. There again, the Doppler effect has brought us the solution of the problem.

The stars, like the Sun, exhibit terrestrial lines (which are the enemies of astronomers) and stellar lines. No one would have imagined that there could be anything else. Yet the light that we receive from a star has made two journeys, relatively very short, across two atmospheres (stellar and terrestrial) and one very long journey across interstellar space. The times of passage are counted by centuries across space, by tenths of a second across the atmosphere of the star and by thousandths of a second across the terrestrial atmosphere.

It is again the Doppler effect which has enabled the interstellar lines to be detected. In the spectrum of a star the displacement of the lines gives the radial velocity, and all the lines should give the same velocity. If a line gives a velocity clearly different from the others—above all, if this velocity does not participate in the variations in velocity which is shown by other lines—it must be produced by matter situated elsewhere than in the star, on the path between the star and ourselves.

The first of these *stationary lines* in a star of variable radial velocity was discovered by Hartmann in 1904 in the spectrum of δ Orionis, of which the great variations in velocity had been discovered by Deslandres in 1900. The velocity, measured on the hydrogen lines, varies between $+133$ and -66 km./sec.; the lines of ionised calcium (Ca^+) (H and K lines)

show a constant velocity of +16 km./sec. Further, the calcium lines have not the same appearance as the others ; the lines of the Balmer series are wide and diffuse, while those of Ca^+ are narrow and sharply defined. It is evident that between the star and ourselves there must be interposed a "cloud of ionised calcium."

Previously, in 1901, Nova Persei had shown a peculiarity which ought to have been interpreted in the same way. While almost all the lines exhibited very variable aspects, with important changes in wave-length (variable velocity of the absorbing gases?), the K line was narrow and of unchanging appearance, showing a constant velocity of +7 km./sec. In turn, numerous other stellar spectra were discovered having lines of calcium independent of the star. The most striking examples are those of the spectroscopic binaries having great variation of radial velocity ; for example, Slipher in 1909 announced the case of β Scorpii, where the variation in radial velocity amounts to 240 km./sec. ; the K line is narrow and stationary.

In the end the idea of the existence of ionised calcium atoms throughout the whole of space impressed itself. The Ca^+ lines are evident in a great number of stars of types Bo to B3, which do not show any stellar calcium ; for later types Ca^+ lines exist in the star itself, they are wide and they mask the interstellar lines. On the other hand, the B stars are of large absolute brightness ; reduced to equal magnitude they are more distant than the majority of the other stars, and consequently the interstellar absorption is stronger.

For fifteen years the Ca^+ lines were the only interstellar lines to be known with any certainty. In 1919 Miss Heger definitely established the existence of lines of neutral sodium (D1 and D2, $\lambda\lambda$ 5890 and 5896). More recently other interstellar lines have been discovered, due to the improvement in spectroscopic methods. Adams and Dunham have discovered the ultra-violet lines of sodium 3302·38 and 3302·98 (the second doublet of the principal series) and two lines of ionised titanium, 3241·99 and 3383·78. This exploration of the ultra-violet region has been rendered possible by the use of aluminised mirrors and of gratings ruled on a layer of the same metal.

Again, Dunham has found the line of neutral potassium 7699·01 (one of the lines of the first doublet of the principal series, of which the other element is hidden by the terrestrial band A). Dunham also finds a line 4226 attributed to neutral calcium.

Finally, other unidentified lines have been discovered, principally by Merrill and his collaborators. They are of a different character from the lines previously discovered ; they appear wider. They are perhaps the heads of unresolved bands. This appearance makes one think of the spectra of molecules ; tentative identifications have been made, but without any certain result. One might also make the very uncertain hypothesis of emission by microscopic crystals which, in the very low temperatures of space, might give absorption bands, just as they are known to emit sharply defined bands (Vegard).

Distribution and Velocity of Absorbing Atoms.—If the absorbing atoms

are uniformly distributed throughout the whole of the galaxy, the intensity of the lines in the different stars should increase regularly with their distance. The examination of this question presents two difficulties: (1) it is necessary to define and measure the intensity of the absorption lines ; (2) the distance of the further stars must be estimated.

The definition and measurement of the intensity of absorption lines are not too easy of accomplishment. The most precise definition is by "equivalent width," defined by the width of a band of complete absorption which absorbs just the same amount of light as the line which is being studied. The measurement of this quantity is not easy, particularly with the low dispersions available for work on stellar spectra.

Struve makes use of an arbitrary scale on which the line intensity is estimated from 0 to 10, by comparison with stellar spectra in which the K line is more or less intense.

In comparing the intensities of the lines of *Na* and Ca^+ a very close correlation of the intensities of the two kinds of line is found ; the distribution of the two atoms *Na* and Ca^+ in space is approximately the same. Absorption by *Na* appears to be a little stronger than by Ca^+. Further, the comparison of the line intensities with the estimated distances of the stars shows an absorption which definitely increases with distance.

Finally, measurement of the exact wave-length of the lines gives us an estimate of the mean radial velocity of the interstellar matter in the path between the star and ourselves. The velocities thus measured relative to the Sun are not very great, they are much less than those of many stars. They would seem to be in agreement with the hypothesis which would attribute to interstellar gas the mean rotation of the galaxy.

Nevertheless it is difficult to believe that radial velocity is the same for the whole length of the light's passage to us. A very recent paper by C. S. Beals * shows that, in certain cases, one can detect masses of interposed gas having different velocities. In analysing the structure of the lines of interstellar sodium in the stars ϵ Orionis, ζ Orionis and ρ Leonis he finds a complex profile, which can readily be interpreted on the supposition of clouds or condensations of interstellar matter having different velocities. The profile of the K line leads to similar results. We have here the beginning of a piece of research the development of which will demand considerable resources.

Let me emphasise the extraordinarily small amount of matter which suffices to produce an intense absorption line ; rather than having recourse to a theoretical calculation, if I recall a well-known experiment it will give you some idea of the amount. The absorption lines of sodium may be obtained very simply in the laboratory by using the vapour of pure sodium. For this, one places a piece of sodium in a glass globe 5 or 6 cm. in diameter ; the globe is exhausted and sealed ; it is then heated in a furnace to slightly above 300° C. The globe then contains sodium vapour at a very low pressure, less than 1/10,000 of an atmosphere. A beam of white light passed through the globe clearly shows in the spectroscope the double

* *Astrophysical Journal*, 1938 June.

dark line of sodium, probably as strong as that produced by interstellar absorption in the spectrum of a very distant star. Imagine this vapour at atmospheric pressure ; its thickness would probably be less than 1/100th of a millimeter (1/2500 of an inch). If this thickness could be expanded along a cylinder of uniform section until it occupied a length of some thousands of parsecs one could get an idea of how much neutral sodium there is in interstellar space.

Theoretical Considerations.—A difficult question presents itself here. Why do the lines of Ca^+ appear relatively strong and those of neutral calcium very weak? Why does sodium, on the contrary, show itself in the form of neutral atoms?

These questions were examined in 1926 by Eddington in a celebrated memoir ; this paper is so well known that I can confine myself to a very brief summary of the results.

Dynamical considerations point to there being a limit to the density of interstellar matter. A possible density would be 10^{-24} grammes per cubic centimetre. Evidently this is a very low density compared with that of our solid or gaseous bodies ; it would correspond roughly to about one kilogram in a volume equal to that of the Earth. In this attenuated medium, which must contain very diverse elements, absorbing atoms can only exist in very small proportions. These atoms must be largely ionised, the ionisation being produced by the radiation of the stars, which penetrates the whole galaxy. In the centre of the galaxy this radiation must be almost uniform in direction ; the density of energy may be $7{\cdot}7 \times 10^{-13}$ ergs per cubic centimetre ; a volume equal to that of the Earth would contain an amount of radiant energy equivalent to 20 kilowatt-hours. The temperature of a black body isolated in space would be, under the action of this radiation, 3° K ; but isolated atoms, absorbing part of this radiation, would have a mean velocity far greater than the speed corresponding to that temperature.

In these conditions the atoms will be largely ionised. Most of the calcium atoms will be doubly ionised (Ca^{++}). The absorption lines of this atom are not within the observable region of the spectrum. A small proportion of Ca^+ atoms exists, and these are the absorption lines which are the most easily observed. As for neutral calcium, it exists only in an extremely small proportion. Sodium is chiefly in the state of Na^+ ; the D lines are produced by the neutral atom, although this is only found in a small proportion.

Continuous Absorption in the Spectrum.—This absorption is caused in varying degrees by all the bodies present in space, from the smallest (electrons) to the largest (asteroids). The atoms which produce absorption lines must also produce a general absorption by diffusion.

This absorption effect can show itself in two different ways:

(1) General absorption of the total radiation, leading to a more rapid increase in apparent magnitude than is indicated by the law of diminishing brightness expressed as $1/r^2$.

(2) Further, the absorption, while continuous in the spectrum, may be

selective ; the result is a change in the composition of the light, revealing itself by a change in *colour*. An increase of absorption with decreasing wave-length is always to be expected, and consequently a *reddening* when the distance increases. In each case the absorption may be expressed as a function of the wave-length.

In most cases the loss of light is not caused by *true absorption*, but by *diffusion* ; the diffused light (to which subject we shall return later) is more or less polarised.

The following table shows the effect of the various particles, arranged in order of increasing magnitude :—

Particles	Law of Absorption in the Spectrum	Polarisation of the Diffused Light
Electrons . . .	Constant	Total polarisation.
Atoms and molecules .	Law of λ^{-4}	Almost total polarisation.
Particles much smaller than a wave-length . .	Do.	Do.
Particles of the order of a wave-length . .	Law of λ^{-n} ($n < 4$; possibly $n = 1$)	?
Large particles . .	Constant (?)	Non-polarised.

Let us examine in turn the two effects indicated above.

(1) *Increase in Apparent Magnitude.*—Let M be the absolute magnitude of a star, m its apparent magnitude and r its distance expressed in parsecs. We have the relation

$$M - m = 5 - 5 \log r - \alpha r,$$

where α is the mean coefficient of absorption of the space traversed, expressed in magnitude per parsec.

For another star we shall have a similar equation. Denoting by dashed letters the relative quantities, and taking the coefficient of absorption to be the same, we shall get—

$$M' - m' = 5 - 5 \log r' - \alpha r'.$$

From a study of the spectra one may find good reasons for believing that the two stars are identical, and consequently that $M' = M$.

From the two equations we have

$$m' - m = 5 \log (r'/r) + \alpha(r' - r);$$

α can be calculated if the distances are known. That is the difficult point. There would appear to be absorption to the extent of perhaps 0·5 mag. per kiloparsec.

Another method of attacking the problem has been suggested by Hubble and applied also by Mineur. It consists in making a count of the extra-galactic nebulæ at different galactic latitudes. It is assumed that these nebulæ are, on the average, uniformly distributed in direction, and that the apparent diminution in their numbers as one approaches the galactic plane is due solely to the absorption in the region of the galaxy.

Unfortunately photometric data on the extra-galactic nebulæ are very meagre and we are obliged to rely on counts. Mineur finds that the best representation of matter in galactic space is to look on it as bounded by two planes parallel to the galactic plane, the total thickness of this layer being 750 parsecs. The Sun would be at about 100 parsecs to the north of the median plane ; when passing perpendicularly through the total thickness the absorption would betray its presence by an increase in photographic magnitude of 0·3. This result is not contradictory to the one obtained above.

Nevertheless it is evident that the distribution of absorbing matter within the galaxy is not uniform. The existence of *dark nebulæ*, which hide everything behind, is the proof that in certain regions there is absorbing matter of exceptional density.

(2) *Selective Absorption.*—From one point of view this is an easier matter to study than general absorption. Knowledge of distances is unnecessary for its estimation ; knowledge of the distances is only necessary for calculating the effect on the unit of distance. Selective absorption has the effect of modifying the spectral intensity curve ; the absolute values do not enter into the question.

If one compares two stars of the same spectral type, one can assume that they emit radiations having the same spectral intensity curve ; if any difference is noticed, in all probability it is that the two stars are at different distances and the selective absorptions have differed. These differences in the spectral curves may show themselves by various observable effects.

(*a*) The most familiar effect, depending on the curve of spectral intensity, is that of *colour*. We say that Antares is red, Capella yellow, Sirius white, Vega blue ; this indicates some differences in the spectral intensity curve, but not with sufficient accuracy for our present problem. To tell the truth, astronomers have never made any very precise visual colour measurements such as illuminating engineers have done ; yet the matching of colours can be done with great accuracy on uniform surfaces, and it is easy to transform the radiation coming from a point-source (a star) into a uniform field. Finally, one may say that the method of visual colour measurement has not given us any information in the problem of selective interstellar absorption.

(*b*) The usual method of detecting the difference of spectral composition is by measuring the *colour index*. It is true that this idea is conventional, but nevertheless it has been of great service. One can define a large variety of "colour indices." The principle is as follows :—

There are many different kinds of apparatus which record radiations—that is to say, apparatus which, receiving a radiation, register a response which increases with the intensity of the radiation received : thermopile, eye, photographic plate, photo-electric cells of various kinds, etc. These recorders are variously "selective"—that is to say, they are not equally sensitive to different radiations of equal energy. Each of them enables us to establish a scale of *magnitudes*, with the usual conventions (a difference of 5 magnitudes indicates a difference of 100 times the light). For radiations of the same spectral composition all these scales coincide ; if one establishes

the agreement for one star, it is established for all the others. But for stars giving different radiations this agreement no longer exists. The "colour index" is the difference of magnitudes given by the two instruments, the common origin being chosen from a star previously determined for the purpose. The difference is always taken in the sense that the colour index is positive for the red stars. It will be seen that one can define a large number of colour indices by combining the indications of *two* receivers having different curves of spectral sensitivity. Nevertheless it is important to choose receivers having plainly defined sensitivity curves and not subject to variation due to improvement in process of manufacture. The thermopile is the best well-defined receiver (equal sensitivity to equal energy for all radiations), but its sensitiveness is insufficient for most astronomical purposes. The normal eye is also very well defined, and appears not to have changed since the beginning of historical time. The photographic plate presents very great advantages, and its sensitivity curve can be modified by process of manufacture or by means of filters ; but astronomers use plates prepared for other purposes, and are obliged to choose from plates offered commercially. The manufacture varies from year to year ; there is nothing to show that the plates in use to-day will still be in use in ten years' time. Finally, there are in existence many varieties of photo-electric cells with differing curves of sensitivity. But here again these curves depend on details of manufacture. I have heard it said by an expert in photo-electric cells that he had never come across two cells that were absolutely identical.

It is well known that the most frequently used colour indices result from the following combinations :—

Eye—non-stained photographic plate.*
Orthochromatic photographic plate with filter—non-stained plate.
Various photo-electric cells.

Often the differences of spectral curve are expressed in terms of *temperature* (more exactly, *colour temperature*, used also in the technique of illumination) ; this introduces one more element of uncertainty.

(*c*) It is only within the last few years that astronomers have made up their minds to use spectrophotometry in the way that physicists have long used it. This leads to registering the spectral intensity curves, or at least to comparing the curves one with another. The most comprehensive way is to make a comparison, for each star, with an artificial source previously studied in the laboratory and serving as a standard curve ; this is what is now being done by some astronomers, and particularly by a group of young astrophysicists (Arnulf, Barbier, Chalonge, Mlle. Safir) who use for their source of comparison a tube of hydrogen giving a continuous spectrum which is constant and well known. But it is a fact that the majority of astronomers do not like artificial sources ; they prefer making comparisons between the stars. I shall come back in a moment to a discussion of the results.

Up till now it is by the method of colour index that the largest number

* These plates are disappearing from commercial manufacture.

of observations has been made, principally on stars of earliest type. One may assume that stars of type O, Bo, B1–B5 have the same intrinsic colour ; without absorption they would have the same colour index, which one may call "normal colour-index." The difference between the observed index and the normal index is called "colour-excess" ; it represents the effect of selective absorption. By this means it is found that there is a selective absorption which is stronger for blue than for red, and increasing with distance.

According to Oort, this selectively absorbing "dust" is very concentrated towards the galactic plane. The thickness of the region where the selective absorption is appreciable would not be more than 300 parsecs. If one takes the differential absorption between wave-lengths 4260 and 4770 it would be about 0·032 mag. for the semi-thickness of the absorbing layer (150 parsecs). Beyond this layer the selective absorption would be negligible up to very great distances. The *gas* producing the lines of absorption (Ca^+ and Na) seems much less concentrated.

These results show an absorption increasing from the red to the violet ; they cannot in any way show us the law of absorption as a function of the wave-length. Only the spectrophotometric measurements allow us to attack this part of the problem. The results are as follows :—

Suppose one has compared, spectrophotometrically, two stars. For each wave-length one can calculate the ratio y_1/y_2 of the ordinates of the spectral intensity curves ; each of these quantities is measured with an arbitrary unit, and their mutual relation is only known to an arbitrary constant factor ; consequently its logarithm, $\log (y_1/y_2)$, is known to an arbitrary constant. This quantity, $\log (y_1/y_2)$, is a function of the wave-length λ. The simplest way of showing the variation is to draw the curve which gives $\log (y_1/y_2)$ as a function of $1/\lambda$. This curve, in most cases, is very nearly a straight line.

$$\log (y_1/y_2) = A/\lambda + B,$$

A and B are two constants, the latter an arbitrary one. The constant A indicates the difference between the two radiations.

For stars of the same type, without absorption, one would have $A = 0$. Now experience shows that for two stars of the same type but at different distances A is not zero ; there is an absorption increasing from the red to the blue, as indicated by the observations by means of the colour index. Further, the curve is approximately a straight line ; that indicates a selective absorption, varying as λ^{-1}. One can conclude from this that the absorbing medium is not formed principally of atoms or of molecules or of very small agglomerations of molecules, which would give a law of absorption of λ^{-4} ; the absorbing "dusts" are combinations which are neither very small nor very large relative to a wave-length.

Study of Light by Reflection or by Excitation.—This is perhaps the most powerful method, with that of the absorption lines, of investigating the nature of interstellar matter. Emission of interstellar matter is shown most plainly in the "gaseous nebulæ." These nebulæ are of two kinds:

one kind gives a continuous spectrum (Pleiades nebula), the other a bright-line spectrum (Orion nebula). In both cases the emitted light is connected with the presence of certain stars visible in the same part of the sky and certainly situated in the midst of the luminous gas.

(1) *Nebulæ with Continuous Spectra.*—The spectrum is the same as that of the exciting stars; the light of the nebula is nothing but reflected (or rather diffused) light, whence the name of "reflecting nebulæ." The reflecting nebula obscures the stars beyond it; one can assume that it is identical with a "dark nebula," and only differs from it in the presence of one or more stars of large absolute brightness in its midst. We need to study the following points about this light: its polarisation, and its colour as compared with that of the exciting star. The results of the observations have been discussed by Struve, who has drawn important conclusions from them. In the first place, the diffused light is *bluer* than that of the star; the law of diffusion may be a law of $\lambda^{-n}(n>0)$. The difference of colour index between the star and the nebula is in the neighbourhood of 0·25 magnitude, and this may give an idea of the value of the exponent n. The law λ^{-4} would suit if the diffusion were produced by atoms or molecules; but this law would give a difference in colour index equal to 1·1, far too large. The law of λ^{-1} would appear to suit.

On the other hand, polarisation is observed, but only partial; the proportion of polarised light rarely exceeds 12 per cent., whereas it would be almost complete for molecular diffusion.

From all this we get the result that the diffusing medium is formed of particles much larger than molecules without being very large in relation to the wave-length of light. Struve concludes that the majority of particles have a diameter between 0·1 μ and 10 μ.

(2) *Nebulæ with Bright-line Spectra.*—The best-known type is the Great Nebula in Orion. The light has very little continuous spectrum; the spectrum is formed of bright lines, particularly those of *H*, *He*, *O*, *N*. Excitation is produced under the action of radiation of very short wave-length from the bright stars of very high temperature which are situated in the gaseous mass of low density. The nebula reveals at the same time the presence of certain atoms and of certain categories of photons.

(3) *General Light of the Sky.*—Side by side with the light of the nebulæ it is necessary to study the general illumination of the night sky, in which is mingled light from all sources. During the last few years this question has given rise to much research—in England by Lord Rayleigh and his collaborators, in France by Dufay, Cabannes, Garrigue, Gauzit, Dejardin, Arnulf, Bernard, almost all of whom are former students of mine.

The starting-point is a commonplace observation: at night, when the sky is cloudless, it is not dark. On a horizontal plane the illumination is about 3×10^{-4} lux, equal to that produced by a 25-candle-power lamp at 300 metres. Study of this light shows that its origin is complex. It consists of a continuous spectrum and of a complicated bright-line spectrum. It may arise from the following causes :—

(1) Direct light from the stars: continuous spectrum, non-polarised light.
(2) Star light diffused by interstellar matter by a process similar to that of the reflecting nebulæ. This cause gives to the sky uniform brightness with a continuous spectrum.
(3) Sunlight, diffused by the medium which may exist in our planetary system. This cause would appear to explain the zodiacal light, but only enters very slightly into the light of the real night sky.
(4) Light emitted by the upper atmosphere (line spectra).

Of all these sources of luminosity, that which most directly concerns our subject is the second (diffusion of the light of the stars). The question has been studied by Dufay and Wang Shi-Ky, who assume that the interstellar medium diffuses without real absorption. Assuming reasonable characteristics for this medium and supposing that it diffuses without true absorption, Wang Shi-Ky finds that on the Earth the light due to diffusion may amount to half the total light arriving directly from the stars. This does not alter the total amount of light received by the Earth, but partly explains the brightness of the sky between the stars.

As for the line spectrum, it is for the most part produced by the upper atmosphere; it does not enter into our discussion. But if there were any emission by interstellar matter it would be mixed with that of the atmosphere; the separation is not easy. However, the light of the atmosphere must increase in intensity as one approaches the horizon, because the atmosphere is seen through a greater thickness, whilst the light of space must diminish because of the absorption. Lines of Ca^+ have been sought for, but have not been found. But recently lines of *Na* have been discovered in the following circumstances.

Using a spectrograph of great light gathering power (but naturally of low dispersion) Cabannes and Dufay found in the light of the night sky a yellow line, the wave-length of which was estimated at 5894 ± 2. The double line D falls within the limits indicated, but the dispersion was not sufficient to separate the two lines, and the attribution to sodium was uncertain. In 1938, using a Perot-Fabry *étalon* Cabannes and Dufay obtained double interference rings, coinciding exactly with those of the doublet D; the presence of sodium lines in the spectrum of the light of the night sky is thus demonstrated. At the same time Bernard showed that this line is greatly intensified at twilight, when the upper atmosphere is still illuminated by the Sun. The sodium doublet is emitted by the very high layers of the atmosphere.

Several questions remain in suspense: What is the origin of this sodium? Does it come from the Earth, where small crystals of *NaCl* exist in abundance, crystals which would have to be dissociated by some unknown process, or is it interstellar sodium gathered by the Earth in its movement across space? What is the origin of the excitation of these sodium atoms? Is solar radiation capable of producing this excitation? These questions remain to be answered.

Absorption in the Interior of the Spiral Nebulæ.—Everything existing in our galaxy seems to exist also in the spiral nebulæ, which resemble our galaxy viewed from the outside. One finds therein variable stars, novæ, super-novæ, clusters. There is reason to believe that interstellar matter also exists there, producing the same phenomena as in our galaxy. Indeed one can even see, in the spirals, dark streaks which may be the equivalent of "our dark nebulæ." This appearance shows that the distribution of this absorbing matter is far from being uniform.

The study of the spirals introduces another important problem: Does any appreciable absorption exist in the space between the galaxies? It is generally assumed that no such absorption exists, and that this part of space is truly empty. I do not know if the evidence for such a state of affairs is complete. As I have said above, photometry of the extra-galactic nebulæ is somewhat imperfect. In any case, if absorption exists it is, for equal distances, incomparably less than in our own galaxy; if it were otherwise, at the enormous distances which separate us from the spirals not one of them would be visible.

Detection of Celestial Objects by Collision with the Earth.—Lastly, a certain number of completely invisible small bodies are detected only at the moment when they encounter the Earth. These are meteors, bolides, aeroliths. They existed in space before encountering the Earth, and certain of them formed a part of interstellar space. By far the most numerous of these little objects are the meteors. Some of them are connected with comets, and form part of the solar system; but the majority of telescopic meteors, which are far more numerous, encounter the Earth with a speed greatly in excess of parabolic velocity; they do not belong to the solar system, and form part of interstellar space. Statistical data of their encounters with the Earth can give us information about their distribution in space. Unfortunately we know hardly anything about their mass, nor even about the cause of their luminosity, when they arrive in our upper atmosphere As for the bolides and aeroliths, these are really small celestial bodies; it would be rash to say anything about their origin.

I must end by apologising for having chosen a subject so vast that I have been obliged to confine myself to a classification of methods of investigation and to an indication of the conclusions which it has been possible to draw from them. The majority of them are as yet only at the first stage of their use. More and more powerful methods, as well as an ever closer association with Physics, are necessary to get from them fresh results. I believe that in a very few years our present state of knowledge will appear prehistoric.

THE HYDROGEN LINES NEAR THE BALMER LIMIT.

A. Pannekoek, Assoc. R.A.S.

1. It is well known that in celestial spectra only a limited number of hydrogen lines of the Balmer series is visible; near the series limit they merge and at last are effaced, a continuous spectrum making its appearance a good deal on the long wave-length side of the theoretical limit. In the spectrum of ordinary A-type stars not more than 20 lines can be distinguished; Menzel gives 29 lines (up to H31) for the chromospheric spectrum.

The disappearance of the higher members of the Balmer series is not exclusively due to their merging; other influences are at work. H. P. Robertson and Jane M. Dewey * have treated the disturbance of the higher electron orbits by the electric fields due to free electrons or ions coming near to the atom. The disturbed orbits cannot be quantized, and the transition to or from the second level takes place in the form of continuous absorption or emission. Thus line radiation is changed into continuous radiation.

The theory of this effect is treated in the same way as the theory of the Stark effect. We introduce parabolic co-ordinates ξ and η, defining a point in a meridian plane containing the direction of the electric field by the parameters of two parabolas intersecting at that point. The 3rd co-ordinate is the equatorial angular co-ordinate ϕ. Then we have the quantum conditions

$$\left.\begin{aligned} &\int p_\phi d\phi = 2\pi p_\phi = n_3 h, \\ &\int p_\xi d\xi = \int \frac{d\xi}{\xi} \sqrt{-p_\phi{}^2 + 2a_1\xi + 2m\bar{W}\xi^2 - meF\xi^3} = n_1 h, \\ &\int p_\eta d\eta = \int \frac{d\eta}{\eta} \sqrt{-p_\phi{}^2 + 2a_2\eta + 2m\bar{W}\eta^2 + meF\eta^3} = n_2 h, \end{aligned}\right\} \qquad (1)$$

where $a_1 + a_2 = 2me^2$, $\bar{W}$ is the (negative) total energy of the electron, and F is the field strength. In order to reduce these expressions to dimensionless numerical values, we express the co-ordinates ξ, η in terms of the radius a_0 of the lowest orbit as unit. Then replacing a_1 and a_2 by $2\gamma_1 me^2$ and $2\gamma_2 me^2$ ($\gamma_1 + \gamma_2 = 1$), and considering that $p_\phi{}^2 = n_3{}^2 h^2/4\pi^2$, $me^2 a_0 = h^2/4\pi^2$, $2m\bar{W}a_0{}^2 = -h^2/4\pi^2 n^2$, and $mea_0{}^3 = (a_0{}^2/e)h^2/4\pi^2$, which we denote by $Eh^2/4\pi^2$ ($E = 5{\cdot}846 \times 10^{-8}$), we find

$$\left.\begin{aligned} &\frac{h}{2\pi}\int \frac{d\xi}{\xi} \sqrt{-n_3{}^2 + 4\gamma_1\xi - \frac{1}{n^2}\xi^2 - EF\xi^3} = n_1 h, \\ &\frac{h}{2\pi}\int \frac{d\eta}{\eta} \sqrt{-n_3{}^2 + 4\gamma_2\eta - \frac{1}{n^2}\eta^2 + EF\eta^3} = n_2 h. \end{aligned}\right\} \qquad (2)$$

When there is no electric field the integration can be easily performed and results in the relations $\gamma_1 = (n_1 + \frac{1}{2}n_3)/n$, $\gamma_2 = (n_2 + \frac{1}{2}n_3)/n$; hence $n_1 + n_2$

* *Physical Review*, 31, 973, 1928.

$+n_3 = n$. In the case of an electric field the term with F must be compensated by a change in $\overline{W}$, determined by $(n_2 - n_1)n$; this is the ordinary Stark effect. Hence in our expressions (2) the coefficient of ξ^2 or η^2, coming from $\overline{W}$, is not exactly equal to $h^2/4\pi^2n^2$, but deviates from this normal value by the amount $2ma_0^2\Delta\overline{W} = 3EF(n_2 - n_1)nh^2/4\pi^2$. In our first approximation we may neglect this deviation and make use of equations (2). Denoting the 3rd degree expressions under the root by $f(\xi)$, $f(\eta)$, the integration extends over the positive range of f, between the limits where $f = 0$. In fig. 1 such curves are drawn ; the parabola $F = 0$ denotes the curve without field for a certain γ ; $\xi_1\xi_2$ and $\eta_1\eta_2$ are the limits for the ξ and the η curve.

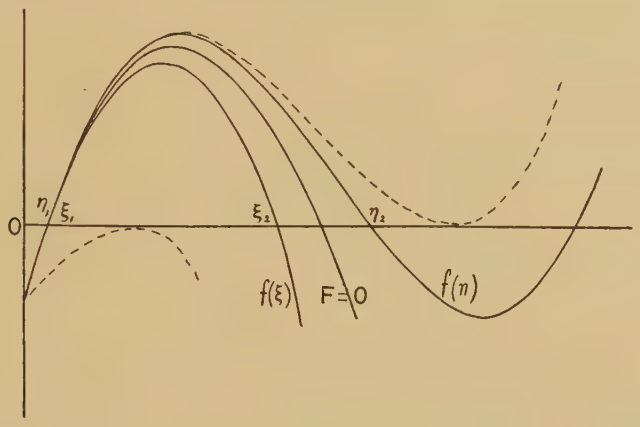

FIG. 1.

This holds as long as the positive part of the f curves is limited by two zero points. As soon as this condition ceases to be fulfilled, the orbits will degenerate. This can take place in two different ways, indicated by the dashed curves in fig. 1. Firstly, when by the increase of F the $f(\eta)$ curve rises so much that its minimum remains above the zero line; then the integration from η_1 extends to ∞, the orbit cannot be quantized any more and the line radiation changes into continuous radiation. Secondly, when by the increase of F the $f(\xi)$ curve falls so low that its maximum remains below the zero line ; then the intersection points $\xi_1\xi_2$ disappear and no ξ-integral is possible. So we find two limiting conditions for the existence of quantized orbits : one that $f(\eta)$ should have a minimum < 0, the other that $f(\xi)$ should have a maximum > 0. Expressing them by means of the coefficients of (2) we find :

$$\left.\begin{aligned} &-\frac{2}{27n^2(EF)^2}\left(\frac{1}{n^4} - 18\gamma_2 EF\right) - n_3^2 - \frac{2}{27(EF)^2}\left(\frac{1}{n^4} - 12\gamma_2 EF\right)^{\frac{3}{2}} < 0 \\ &\qquad\qquad (\eta\text{-condition}), \\ &-\frac{2}{27n^2(EF)^2}\left(\frac{1}{n^4} + 18\gamma_1 EF\right) - n_3^2 + \frac{2}{27(EF)^2}\left(\frac{1}{n^4} + 12\gamma_1 EF\right)^{\frac{3}{2}} > 0 \\ &\qquad\qquad (\xi\text{-condition}). \end{aligned}\right\} \quad (3)\ *$$

* These equations correspond to the 2nd and 3rd of equations 13, p. 975, of Robertson and Dewey. Their a_2/X is identical with our $1 - k^2$ of (4).

The numerical computation for the η-condition was made in the following way. We put :

$$\left.\begin{aligned} 12\gamma_2 n^4 EF &= 1 - k^2 ; \\ \text{then} \qquad (-k^3 - \tfrac{3}{2}k^2 + \tfrac{1}{2}) &< \tfrac{27}{2} n_3^2 n^6 (EF)^2 \end{aligned}\right\} \qquad (4)$$

is the condition for line radiation. The term on the right of the inequality is (the extreme case of very strong fields excepted) a small quantity ; hence the dissolution of the line takes place when the left-hand side $\frac{1}{2} - \frac{3}{2}k^2 - k^3$ reaches a small positive value Δ. For k increasing from 0 to 0·5 this function decreases from 0·5 to 0, whereas $1 - k^2$ in the same time decreases from 1 to 0·75. The change therefore takes place for $1 - k^2 = 0{\cdot}75 + \frac{4}{9}\Delta$, only a small amount above 0·75. In first approximation, hence, the dissolution of the line begins for

$$12\gamma_2 n^4 EF = 0{\cdot}75(+\tfrac{4}{9}\Delta). \qquad (5)$$

Introducing the numerical value $12EF = 7{\cdot}016 \times 10^{-7} F$, and solving for γ_2, we find $\gamma_2 = 106{\cdot}9 \times 10^4 n^{-4}/F$. So we have for

	$F=535$	214	107	53	21	6·7 e.s.u.
for $n=10$	$\gamma_2=0{\cdot}20$	0·50	1·0			
$n=20$		0·03	0·06	0·12	0·31	1·0
with	$\Delta=0{\cdot}05$	0·008	0·002 (for $n=10$).*			

In this computation of Δ we took $n_3 = 2$ because transitions between the nth and the 2nd level are only possible for $n_3 = 1$, 2 or 3. The quantity γ_2 denotes a fraction of the total breadth of the Stark pattern of the line ; for $n_3 = 1$ we have $\gamma_2 = 1/2n$ for the extreme red components, increasing to $1 - 1/2n$ for the extreme violet components. All the Stark components with γ_2 below the values computed above from (5) persist as separate lines, whereas the components with γ_2 above these values are dissolved. So we see that, with increasing field strength, first the extreme red components disappear, then gradually the dissolving process proceeds toward the smaller wave-lengths, and the extreme violet components are the last to be attacked. This is in accordance with the experiments of Rausch von Traubenberg,† who saw in a field of nearly $F = 2300$ e.s.u. (= 700,000 volt/cm.) the line $H\gamma$ split up in a bundle of lines which in the inhomogeneous field was cut off obliquely, so that the violet components persisted longer than the red ones.

For the ξ-condition we introduce an analogous substitution :

$$12\gamma_1 n^4 EF = k^2 - 1 ; \qquad k^3 - \tfrac{3}{2}k^2 + \tfrac{1}{2} > \tfrac{27}{2} n_3^2 n^6 (EF)^2. \qquad (6)$$

For moderate fields, where k is a small amount above 1, we find that the transition point is independent of EF and for any field strength is determined by $\gamma_1 = \frac{1}{2} n_3/n$. For this value of γ_1 the function $f(\xi)$ with $F = 0$ has a maximum exactly on the zero line, giving a constant ξ_1 for this orbit. It is clear that

* The condition for the limit of continuous radiation, given by R. and D. (form 21′, p. 977), usually quoted in the simplified form: $|W| < 2e\sqrt{eF}$, corresponds to (5) for the case $\gamma_2 = 1$.

† *Die Naturwissenschaften*, **18**, 417, 1930.

any trace of an electric field brings this maximum below the zero line and destroys the orbit, which is in fact the one giving rise to the extreme violet component of the Stark pattern. The other orbits can be dissolved by the ξ condition only in the case of strong fields, increasing the $(EF)^2$ term above k^3. By elimination of EF we find

$$\gamma_1^2 > \left(\frac{n_3}{n}\right)^2 \frac{3}{32} \frac{(k^2-1)^2}{k^3 - \frac{3}{2}k^2 + \frac{1}{2}}.$$

Assuming, for instance, $n_3/n = 0{\cdot}1$ this leads to the numerical values

$k =$	2	5	10	20
$\gamma_1 >$	0·058	0·078	0·104	0·145
$F =$	7400	44000	140000	400000

These values of F, necessary to destroy only a small part of the pattern, are so large that they have no practical importance for our problem. So the ξ-condition does not affect the visibility of the lines in another way than already indicated for the extreme violet component.

2. It was not necessary in these computations to proceed to a second approximation, because for an exact treatment it is better to make use of the work of C. Lanczos.* Although the subjects of the papers are apparently different—for Robertson and Dewey the origin of the continuous spectrum, for Lanczos the weakening of the lines—they treat, in reality, the same question, the transformation of line radiation into continuous radiation ; and they proceed from the same equations. Their method of treatment, however, is different, one by quantum mechanics, the other by wave mechanics ; the latter allows exact computations of intensities. In the wave-mechanical treatment there is no sharp limit between existence and non-existence of the line ; before the quantum-mechanical limit is reached a kind of pre-dissolution begins to take place, and the existence of the line gradually dwindles after the limit is passed. Lanczos introduces a quantity δ, the rate of dissolution of the orbit in the presence of an electric field, so that the probability of the nth state of energy is given by $e^{-2\delta t}$. This probability must be combined with that dependent on the spontaneous disappearance of the nth state, determined by $e^{-2\alpha t}$, where $1/2\alpha = T$ is the mean lifetime of this state. The dissolution by the electric field is only perceptible if it takes place more rapidly than the normal disappearance, hence for $\delta >> \alpha$. The number of atoms in this state, N, compared with the number in the case of absence of the field, N_0, is given by $N = N_0\alpha/(\alpha+\delta)$. Hence the intensity of the spectral line is

$$I = I_0/(1+\delta/\alpha). \tag{7}$$

The rate of dissolution δ depends on two integrals I_1 and I_2, occurring in the Schrödinger function:

$$\delta = \frac{\pi e^2}{ha_0} \frac{e^{-2I_2}}{2I_1} = 2{\cdot}066 \times 10^{16} \frac{e^{-2I_2}}{I_1}. \tag{8}$$

* "Zur Intensitälsschwächung von Spektrallinien," *Z. f. Physik*, 68, 204, 1931.

The exact computation of these integrals is a very complicated matter, since in strong electric fields the original quantum numbers themselves are distorted into "effective" quantum numbers, which must be found from series that are not convergent if the field strength is too large. It appears, however, that in such moderate fields as occur in ionized gases in celestial atmospheres these higher-order terms are rather insignificant, so that in the approximations used here these distortions of the quantum numbers may be neglected. Then we have to apply the following system of equations, given by Lanczos in his formulæ 51, 52, 53, 54, 58, 61, 64 (pp. 224–226 *loc. cit.*) :

$$\left.\begin{aligned} &\epsilon = \frac{2a_0^2}{e} F n^3 ; \qquad \tau = 8(n_2 + \tfrac{1}{2}(n_3 + 1))\epsilon = \sin^2 \phi, \\ &k = tg\tfrac{1}{2}\phi ; \qquad k' = \sqrt{1 - k^2}, \\ &3\epsilon I_2 = 2 \cos \tfrac{1}{2}\phi \,.\, E' - 2 \sin \phi \sin \tfrac{1}{2}\phi \,.\, K', \\ &\epsilon I_1 / n^2 = 2 \cos \tfrac{1}{2}\phi (K - E), \end{aligned}\right\} \qquad (9)$$

where E and K are the complete elliptic integrals of the first and second kind, and E' and K' are the same for the modulus k'. The factor occurring in τ was written $n_2 + \frac{1}{2}$ by Lanczos and restricted to the extreme components ; the more general form $n_2 + \frac{1}{2}(n_3 + 1)$ corresponds to what was called $\gamma_2 n$ in the former treatment. We have here to consider that the treatment by wave mechanics gives for n_3 the values 0, 1, 2 (for $n = 2$) instead of 1, 2, 3 ; this difference is compensated by the occurrence of $n_3 + 1$ instead of n_3 in (9).

Equation (7) shows that for $\delta >> \alpha$, $I << I_0$, hence the line has disappeared and is dissolved ; for $\delta << \alpha$, $I = I_0$, the line is unchanged. From equation (8) we see that for small I_2 (the variations of I_2, because it is in the exponent, are the most important) δ is large, of a much higher order than α (which for low levels is about 10^8), hence the line is absent. With increasing I_2, δ decreases, and when its decreasing value passes α, then the change from invisibility to normal intensity takes place rapidly : a rather small change in F suffices to bring it about. For the higher levels of hydrogen we may assume the value of 10^6 for α ; the uncertainty as to its exact value has, as will be seen, no material influence. Moreover, it is not necessary to represent the rapid continuous change ; we may replace it by a sudden disappearance at a certain value $\log \delta = \log \alpha$ (which we will take equal to 6), the more so as the results will have to undergo some smoothing process afterwards. From the relations (8) and (9) the value $\delta = 10^6$ determines a certain τ, and so a certain value of $n_2 + \frac{1}{2}(n_3 + 1) = \gamma_2 n$. So, for a given field F and for a certain n the division mark between the visible and the dissolved part of the Stark pattern may be computed.

For this purpose corresponding values of $z = 3\epsilon I_2$ and $y = \log \epsilon I_1 / n^2$ have been computed for different values of τ ; they are collected in Table I.

There is a very smooth variation of $z = 3\epsilon I_2$ with τ ; for practical computations we may represent it by an interpolation formula

$$\tau = 1 - 0{\cdot}60z + 0{\cdot}05z^2. \qquad (10)$$

TABLE I

Values of $3\epsilon I_2$ *and* $\epsilon I_1/n^2$

τ	$z=3\epsilon I_2$	$y=\log \epsilon I_1/n^2$	τ	$z=3\epsilon I_2$	$y=\log \epsilon I_1/n^2$
$1-2\cdot3\times10^{-8}$	$3\cdot8\times10^{-8}$	+0·797	0·8830	0·197	−0·095
$1-1\cdot4\times10^{-5}$	$2\cdot3\times10^{-5}$	·601	0·850	·254	·147
0·99977	0·00041	·479	0·800	·340	·216
0·99880	0·00216	·386	0·750	·426	·279
0·99614	0·0066	·305	0·700	·515	·339
0·99038	0·0160	·230	0·600	·696	·455
0·9847	0·0255	·185	0·5868	·721	·472
0·9796	0·0342	·155	0·500	·882	·574
0·9698	0·0501	·110	0·4132	1·049	·686
0·940	0·101	+0·018	0·250	1·380	·951
0·920	0·135	−0·028	0·1170	1·678	1·313
0·900	0·170	·066	0·0301	1·902	−1·921

TABLE II

Values of A

$3\cdot454\epsilon$	$n=9$	10	11	15	18	20	22	28	30	32
					$\log\alpha=6$					
0·200	7·02	6·92	6·83	6·56	6·41	6·32	6·23	6·01	5·94	5·88
·100	6·75	6·65	6·56	6·27	6·10	6·00	5·91	5·69	5·62	5·56
·050	6·29	6·20	6·11	5·82	5·65	5·55	5·47	5·24	5·18	5·12
·040	6·15	6·05	5·97	5·68	5·51	5·42	5·33	5·11	5·04	4·98
·025	5·87	5·78	5·69	5·41	5·24	5·15	5·06	4·84	4·78	4·72
·020	5·74	5·65	5·56	5·28	5·11	5·01	4·93	4·71	4·64	4·58
·010	5·35	5·26	5·17	4·89	4·72	4·62	4·54	4·32	4·26	4·20
·005	4·97	4·88	4·80	4·51	4·35	4·25	4·16	3·94	3·88	3·82
·002	4·49	4·39	4·31	4·03	3·86	3·77	3·68	3·46	3·39	3·33
					$\log\alpha=8$					
0·250	6·03	5·91	5·81	5·45	5·25	5·14	5·03	4·77	4·70	4·63
·200	5·60	5·49	5·40	5·09	4·90	4·80	4·70	4·46	4·39	4·33
·100	4·89	4·79	4·70	4·41	4·24	4·14	4·05	3·83	3·76	3·70
·050	4·38	4·28	4·20	3·91	3·74	3·65	3·56	3·34	3·28	3·32
·040	4·23	4·13	4·05	3·77	3·60	3·51	3·42	3·20	3·14	3·08
·025	3·94	3·85	3·76	3·48	3·32	3·22	3·13	2·92	2·85	2·80
·020	3·81	3·71	3·63	3·35	3·18	3·08	3·00	2·78	2·72	2·66
·010	3·41	3·31	3·23	2·95	2·78	2·69	2·60	2·39	2·32	2·26
·005	3·03	2·93	2·85	2·57	2·40	2·31	2·22	2·01	1·94	1·88
·002	2·54	2·44	2·36	2·08	1·92	1·82	1·74	1·52	1·46	1·40

For $\tau = 0$, $z = 2$ the real limiting gradient $dz/d\tau$ is logarithmically infinite.

For $\tau = 1$, $z = 0$ the exact limiting gradient is $-4\sqrt{2}/3\pi = -0{\cdot}6002$; for our practical computations this discrepancy is irrelevant. It was not found possible to represent y by an interpolation formula. We have now to solve equation (8) for the point of disappearance, $\delta = \alpha$:

$$16{\cdot}01 - 2 \times 0{\cdot}4343 I_2 - \log I_1 = 6{\cdot}0,$$

or

$$10{\cdot}01 - y + \log \epsilon - \log n^2 = \frac{z}{3{\cdot}454\epsilon}. \qquad (11)$$

To find z, this equation, combined with the relation between y and z as given by Table I, was solved by graphical interpolation for different values of ϵ and n. The results are given in Table II in the form of values of the quantity $A = z/3{\cdot}454\epsilon$, as a function of n and $3{\cdot}454\epsilon$. The computation has been repeated for $\log \alpha = 8$, to see the effect of different assumptions about α.

Now that $z = 3\epsilon I_2$ is known as a function of ϵ, the interpolation formula (10) gives τ ; from τ and ϵ the quantity $\gamma_2 n$ is found by $\tau = 8\gamma_2 n\epsilon$. So we have

$$\gamma_2 n = \frac{1}{8\epsilon}(1 - 0{\cdot}60z + 0{\cdot}05z^2).$$

Inserting the values

$$\epsilon = 1{\cdot}169 \times 10^{-7} F n^3 \quad \text{and} \quad z = 3{\cdot}454\epsilon A = 4{\cdot}040 \times 10^{-7} F n^3 A,$$

we find

$$\gamma_2 = \frac{106{\cdot}9}{F}\left(\frac{10}{n}\right)^4 - 0{\cdot}259\frac{A}{n} + 0{\cdot}0872 \times 10^{-5}\left(\frac{n}{10}\right)^2 A^2 F. \qquad (12)$$

The first term is identical with the result of the former approximate calculation by means of equation (5).

The decrease of the number of visible higher members of a series with increasing electric field has been observed for some potassium series by Kuhn * and by Bakker.† Miss Dewey measured the extension of the continuous spectrum to the red side of the series limit for helium.‡ Chalonge discussed the dissolution of the lower Balmer lines $H\gamma - H\zeta$ into continuous emission in his researches on the continuous hydrogen spectrum.§ All these laboratory experiments with constant fields confirm the theoretical deductions qualitatively, though they do not allow an exact quantitative test of the formulas.

3. In a partly ionized gas we have to deal with variable electric fields, depending on the continually changing distances of the charged particles from the hydrogen atoms. The probability of a field F is given by Holtsmark's function $\bar{W}(\beta)$, where $\beta = F/F_0$, and where F_0, a certain

* *Zs. f. Physik*, **61**, 805, 1930.
† *Proc. Amsterdam*, **36**, 589, 1933.
‡ *Phys. Review*, **35**, 155, 1935.
§ *Ann. de Phys.*, XI, 1, 123, 1934.

mean or normal field strength, depends on the physical parameters of the gas: $F_0 = 2{\cdot}61N^{2/3}e$ (N = number of charged particles per unit volume) $= 46{\cdot}8(2P/T)^{2/3}$ (P = electron pressure). The function $\overline{W}(\beta)$ has been computed by Verwey.* For a certain F or β the pattern of Stark components is extended over a breadth given by

$$\Delta\nu/\nu = \Delta\lambda/\lambda = 3EF\{n(n-1)+2\}\left(\frac{1}{4}-\frac{1}{n^2}\right)^{-1},$$

or

$$\Delta\lambda = \pm\, 0{\cdot}00256\left(\frac{n^2}{n^2-4}\right)^2\{n(n-1)+2\}F = \pm\, SF. \qquad (13)$$

For the lower members of the Balmer series, where the intensity of each Stark component is known, the resulting distribution of intensity over the broadened line could be exactly computed. For the higher members these intensities are not known. Hence we will assume that for each line the total intensity is evenly distributed over a band with a breadth $2B = 2\beta B_0$, where $B_0 = SF_0$. To each value of β (*i.e.* between β and $\beta + d\beta$) belongs the corresponding breadth $2\beta B_0$, and its intensity at each point is given by $\overline{W}(\beta)d\beta : 2\beta B_0$. If there was no dissolution of the Stark components the total intensity for a certain $\Delta\lambda$ would be found by integrating over all values of β, from the lowest $\beta_1 = \Delta\lambda/B_0$, for which the extremity of the band just reaches this $\Delta\lambda$, up to ∞.

$$I = U(\beta_1) = \int_{\beta_1}^{\infty}\frac{\overline{W}(\beta)}{\beta}d\beta\,; \qquad \int_{-\infty}^{+\infty}U(\beta_1)d\beta_1 = 1. \qquad (14)$$

Now a certain part of the red side of each band is dissolved and changed into continuous radiation; this means that part of each band is cut off at a point given by the fraction γ_2 of the total breadth $2\beta B_0$. We may represent these conditions by a diagram (fig. 2), where the abscissa represents $\Delta\lambda$, and β is counted from 0 upward, and the breadth of the Stark band for every β, proportional to β, is included between the two inclined straight limiting lines r and v. Each horizontal line between these limiting lines represents the band for β, its vertical co-ordinate, and must be provided with an intensity value $\overline{W}(\beta)/\beta$. We get the total intensity $U(\beta_1)$, as given by (14), for some $\Delta\lambda$ by integrating along a vertical line, corresponding to this $\Delta\lambda$, the intensity values upward from the point, where the vertical intersects the limiting line, *i.e.* from β_1. Without the dissolution of line radiation the integration would extend to ∞; now, however, the red side part of the whole diagram is cut off by a clipping line, intersecting each horizontal band at a distance $\gamma_2 \times 2B = 2\gamma_2\beta B_0$ from its left-hand end, where

$$\gamma_2 = \frac{107}{\beta F_0}\left(\frac{10}{n}\right)^4 - 0{\cdot}259\frac{A}{n} + 0{\cdot}0872\,.\,10^5\left(\frac{n}{10}\right)^2 A^2\beta F_0,$$

or

$$2\gamma_2 B = 2B_0\left\{\frac{107}{F_0}\left(\frac{10}{n}\right)^4 - 0{\cdot}259\frac{A}{n}\beta + 0{\cdot}0872\,.\,10^5\left(\frac{n}{10}\right)^2 A^2\beta^2 F_0\right\}. \qquad (15)$$

* *Public. Amsterdam*, 5, Table 3, p. 17.

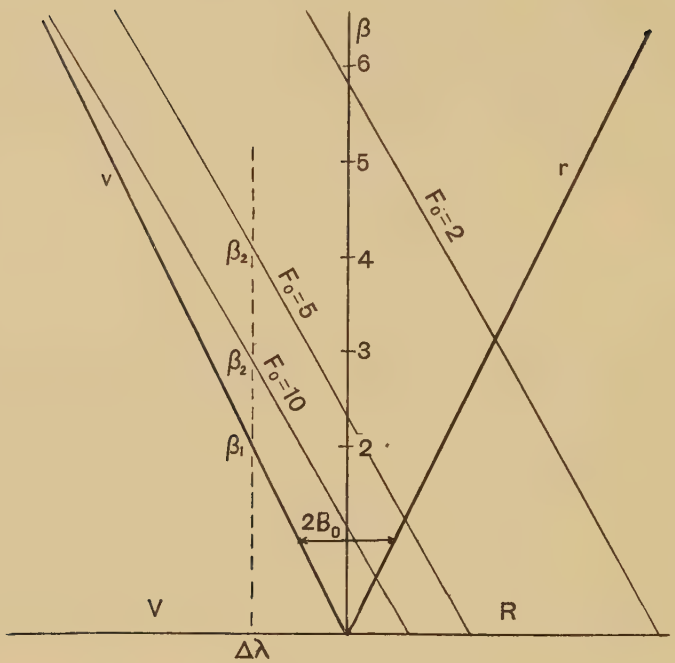
β
6
5
4
3
2
v
r
F_0=2
β_2
F_0=5
β_2
F_0=10
β_1
$2B_0$
V
R
Δλ

FIG. 2.

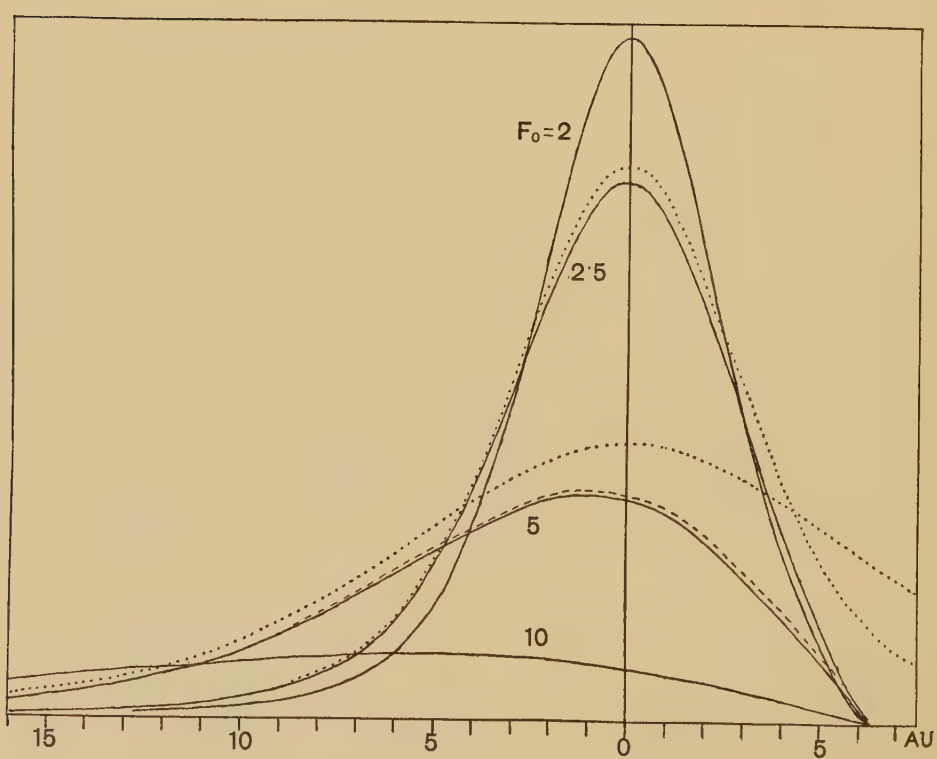
F_0=2
2·5
5
10
15
10
5
0
5
AU

FIG. 3.

For a given value of F_0 (and n) this function of β can be computed; the values of A are taken from Table II, representing A as a function of $\log \epsilon = \log 1{\cdot}17 \times 10^{-7} n^3 F$. Then the clipping lines are drawn in the diagram (in fig. 2, standing for $n = 20$ they are inserted for $F_0 = 2$, 5 and 10), running as nearly straight lines from the red side below to the violet side upward, so that the part dissolved at the red side is at the same time situated at the upper side. For each $\Delta\lambda$ the integration now extends from β_1, not to ∞ but only to the intersection point β_2 with the clipping line. The fraction $\int_{\beta_1}^{\beta_2} : \int_{\beta_1}^{\infty}$ denotes the part of the total brightness left at this point $\Delta\lambda$ of the broadened line.

TABLE III

Intensity Distribution in Balmer Lines

F_0	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6
							$n = 10$								
200						1	40	139	118						
100				2	8	29	110	286	326						
,,			(1	5	12	35	120	304	352)						
70			2	6	14	41	135	344	450	149					
50		1	3	8	19	49	150	376	522	302					
,,		(1	3	9	20	51	153	380	533	322)					
35	2	3	5	10	22	54	158	391	555	373	113				
25	3	4	6	12	23	57	162	397	565	393	152	37			
20	3	5	7	13	25	58	164	399	568	397	160	51	13		
10	4	5	8	14	26	60	166	402	572	402	166	59	25	13	7
							$n = 20$								
10			1	3	9	28	100	239	212						
7		1	2	6	13	38	124	307	341						
5	1	2	4	8	17	46	141	352	455	127					
,,	(1	2	4	8	18	47	144	358	465	144)					
3·5	2	3	5	10	20	52	153	379	527	301					
2·5	3	4	6	11	22	55	159	392	556	371	100				
,,	(3	4	6	11	23	56	160	393	558	376	109)				
2	3	4	7	12	23	56	162	396	563	388	141	12			
1·4	4	5	7	13	24	58	164	399	568	399	164	51	12		
1	4	5	8	14	26	59	165	401	571	400	164	57	22	9	2
							$n = 30$								
2·5			1	4	9	27	93	214	162						
2		1	2	5	12	33	108	256	228						
,,		(1	2	5	12	35	114	270	247)						
1·4		1	3	7	15	40	129	317	356						
1	2	2	4	9	18	48	145	358	466	136					
,,	(2	2	4	10	19	49	147	362	477	150)					
0·5	2	3	5	11	23	56	160	393	558	374	109				
0·35	3	4	7	13	24	58	163	398	567	394	154	38			
0·25	4	5	8	14	26	60	165	401	571	401	164	57	22	10	2

The integrations have been performed, for a number of different values of F_0 for each of the quantum numbers n of Table II, for every tenth part of $2B_0$. An extract of the results for the intensity, for entire multiples of B_0, is given in Table III for $n = 10, 20, 30$. (The lines in parentheses hold for $\log \alpha = 8$.) Each line represents an intensity curve for that n and F_0. The scale of the abscissæ must be transformed into a scale of wave-lengths by means of $B_0 = SF_0\ AU$. As an example, the intensity curves for $n = 20$ are represented in fig. 3. The dashed line for $F_0 = 5$ holds for $\log \alpha = 8$; it shows that errors in the assumed value of α have only a small effect. A dotted line for $F_0 = 5$ and for 2·5 indicates how the original curve without dissolution of line radiation would have run. By this asymmetrical dissolution the maximum and the barycentre of each line are displaced towards the violet side.

The intensity curves for different F_0 are cut off at the red side all at nearly the same distance in AU. This limit is determined in fig. 2 by the intersection point of the clipping line with the right-hand side limiting line. If we assume the former to be exactly straight, corresponding to the first two terms of (15) with constant A, then the intersection point is given by $(107/F_0)(10/n)^4 : 2(1 + 0{\cdot}259A/n)$ in units of B_0; if the scale is transformed into AU by substituting $B_0 = SF_0$, we see that F_0 disappears from the result. We find, by means of these expressions, for the red side limit of the curves:

for $n =$	9	10	11	15	18	20	22	28	30	32	
	28·9	23·6	19·7	10·9	7·7	6·3	5·2	3·27	2·86	2·52	AU,

roughly varying with $1/n^2$.

By integrating the intensity in the tables and curves we find what fraction of the total intensity of the line remains as line radiation, while the other part is dissolved into continuous radiation. The directly computed results are compiled in Table IV.

TABLE IV

Theoretical Intensities of Balmer Lines in Percentages of Normal Intensities

n	$F_0 = 200$	100	70	50	35	25	20	10	7	5	3·5	2·5	2
9	26	59	75	86	93	96	98	99		100			
10	15	40*	57	74*	86	92	95	99		100			
11	8	26	40	56	73	85	90	97		98			
15				14	23	35	45	76	87	93	97	98	99
	$F_0 = 10$	7	5	3·5	2·5	2	1·4	1	0·7	0·5	0·35	0·25	
18	45	61	76	87	93	96	97	99	99	100			
20	29	43	58†	74	85†	90	95	97	99	100			
22	19	29	42	57	72	81	89	94	97	98			
28					33	41	57	72	84	92	96	98	
30					25	32‡	45	60‡	75	86	93	96	
32					18	24	36	49	64	78	88	94	

* 44 and 76. † 60 and 86. ‡ 34 and 62 for $\log \alpha = 8$.

This percentage of residual intensity can be represented rather exactly as a function of $\log F_0 + 3{\cdot}8 \log (n/20)$; for values of this logarithm written in the next line the percentage takes the value of the second line :

$\log F_0\left(\frac{n}{20}\right)^{3\cdot8}$	$-0{\cdot}5$	$-0{\cdot}2$	0	$0{\cdot}1$	$0{\cdot}2$	$0{\cdot}3$	$0{\cdot}4$	$0{\cdot}5$	$0{\cdot}6$	$0{\cdot}7$	$0{\cdot}8$	$0{\cdot}9$	$1{\cdot}0$	$1{\cdot}2$	$1{\cdot}5$
percentage	100	99	97	96	94	91	85	78	68	58	48	37	27	13	6

By means of this little table we can find for every condition of the gas, indicated by F_0, how the residual intensities of the Balmer lines decrease with their increasing quantum number, till they disappear.

4. There is, however, another circumstance to be considered here, the merging of the consecutive members of the Balmer series. If the electric field increases in strength and the Stark pattern broadens, the patterns of adjacent lines at last will eventually overlap. The breadth of the Stark pattern is given by $h\Delta\nu = 3ea_0F\{n(n-1)+2\}$; the distance of two adjacent lines for large n is given by $h\Delta\nu = e^2/a_0n^3$. They are equal for

$$3a_0^2eF\{n^4(n-1)+2n^3\} = 1 \quad \text{or} \quad F = (1/3E)\{n^4(n-1)+2n^3\}^{-1};$$

for $n = 10$, 20, 30 this formula gives $F = 62$, $1{\cdot}9$, $0{\cdot}24$. In this case of overlapping it is not permissible in physical theory to treat the Stark effect of each line separately ; the collectivity of lines must be treated together. In our case the result of this treatment must be smoothed afterwards into a continuous intensity curve produced by continually changing fields. We may assume that the result will be the same, if first we treat each line separately, compute the smoothed result of changing fields, and then combine the results for the different lines.

It is readily seen that lines appreciably dissolved, completely merge into a continuous spectrum of nearly constant intensity ; it is only for much weaker fields where the dissolution is small that the lines begin to be separately visible. The relation between these two causes of disappearance of the lines depends on the principal quantum number ; the merging of adjacent lines depends on Fn^5, the dissolution on Fn^4, hence for the lower members the dissolving process will be more important than for the higher members of the series.

A combined intensity curve of all the lines was computed and constructed for $F_0 = 20$ in the vicinity of $n = 20$, for $F_0 = 2$ and $1{\cdot}4$ in the vicinity of $n = 20$, and for $F_0 = 0{\cdot}2$ in the vicinity of $n = 30$. For the values of n contained in Table II the intensity curves were taken from direct computation ; for the other values they were found by interpolation, using F_0n^4 as the argument. The normal total intensity for each was assumed to be $\sim 1/n$, because in conditions of equilibrium the population of the higher levels increases with n^2, and the transition probabilities approach n^{-3}. The result is shown in figs. 4–6. The separate curves of the spectral lines are given by dots (in fig. 5 they are only given for $F_0 = 2$), the combined curve is given in full (for $F_0 = 1{\cdot}4$ in fig. 5 by a dashed line). We see that with decreasing wave-length the undulations rapidly decrease and disappear, and a continuous radiation remains. Connecting the constant part with

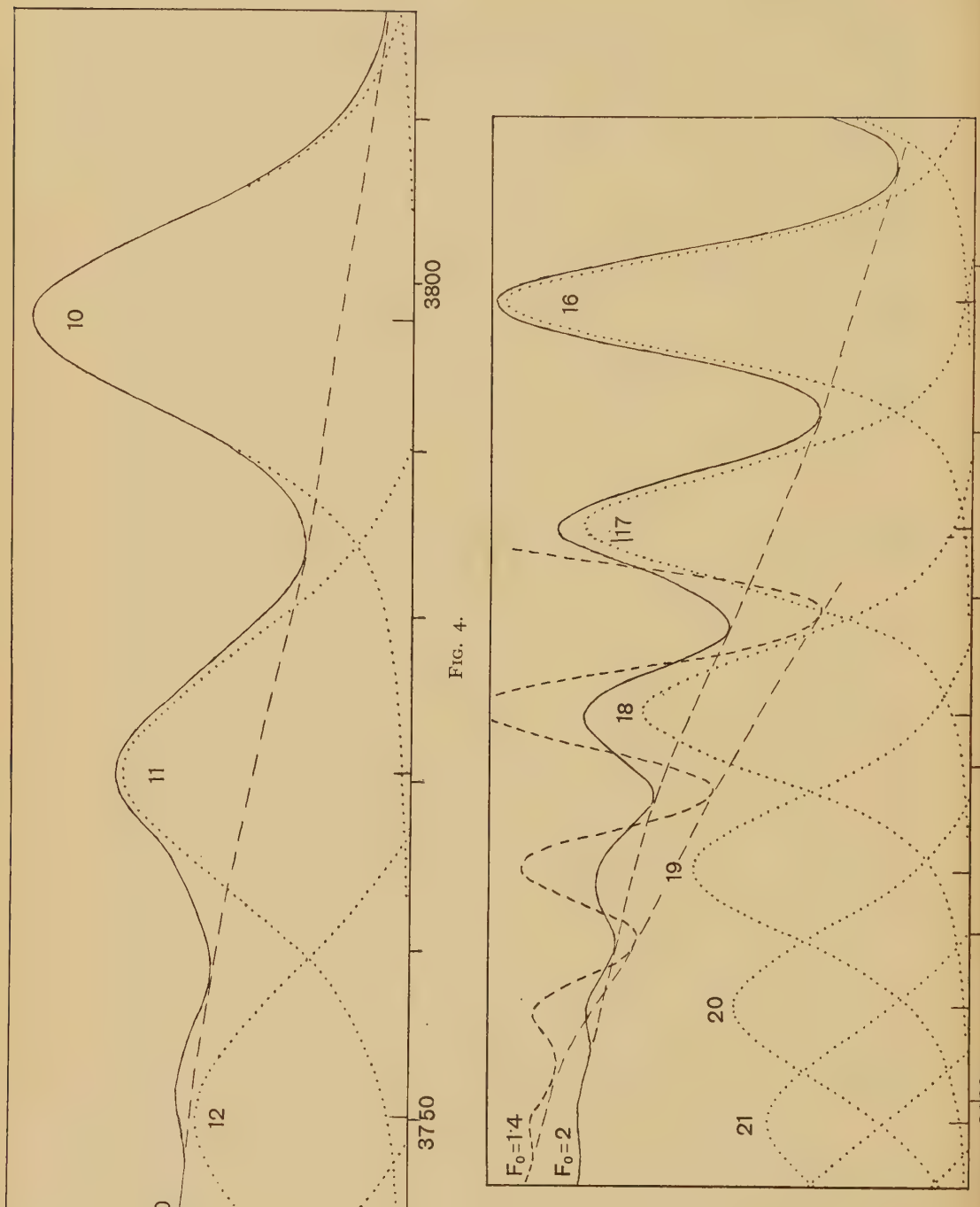

Fig. 4.

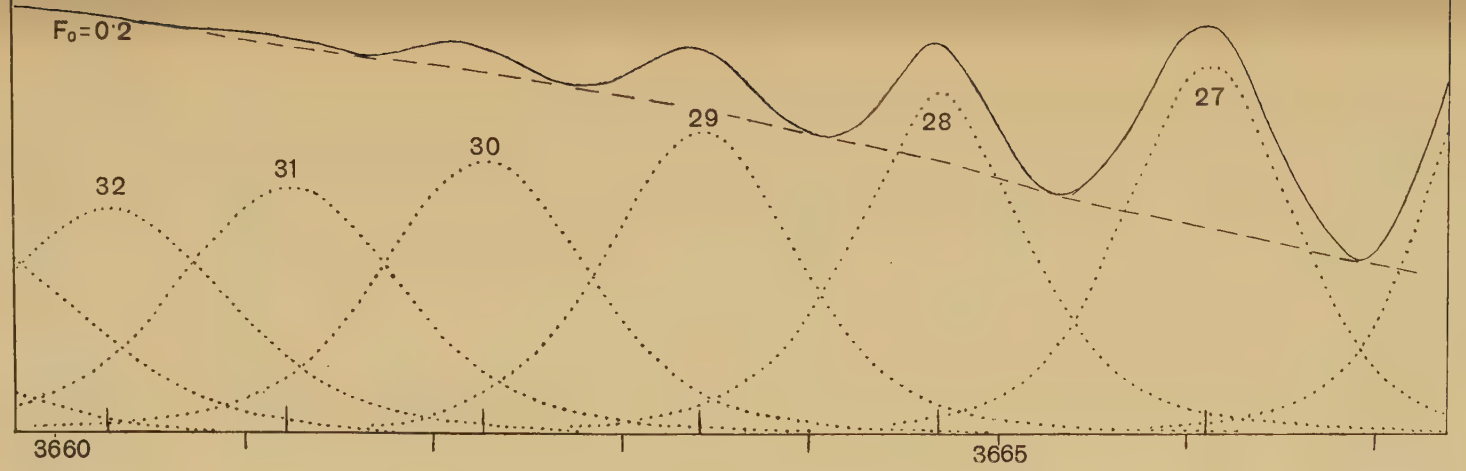
F₀=0·2
32
31
30
29
28
27
3660
3665

FIG. 6.

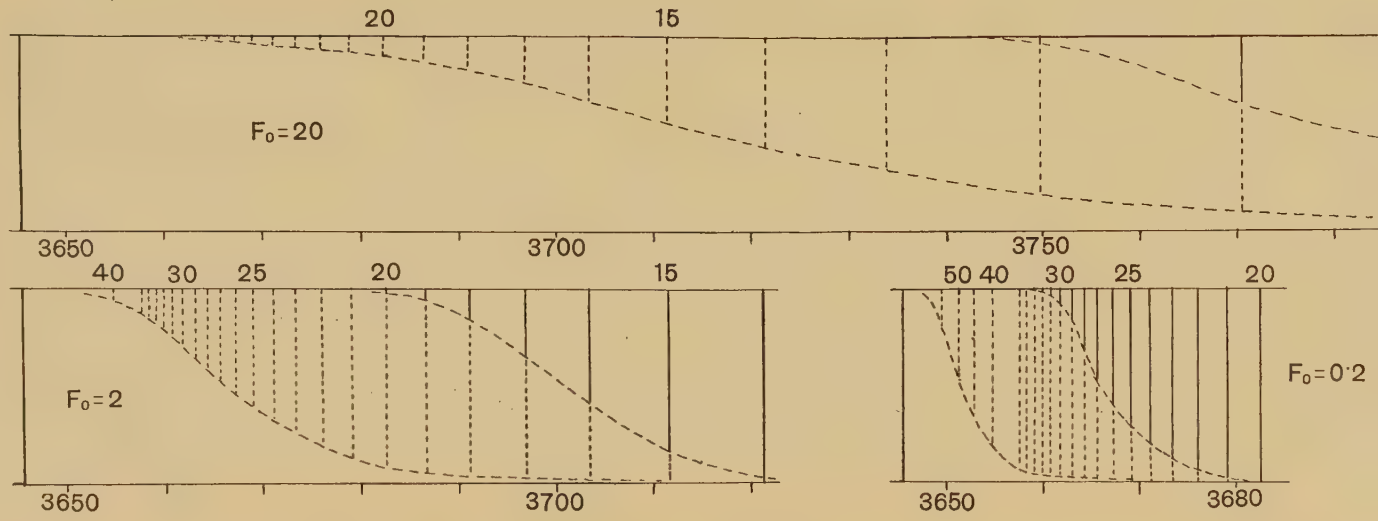
20
15
10
F₀=20
3650
3700
3750
40
30
25
20
15
F₀=2
3650
3700
50 40
30
25
20
F₀=0·2
3650
3680

FIG. 7.

the deepest valley-points in the curve, we see the hydrogen lines rise above a continuous background which decreases for increasing wave-length. What we observe as line intensity is the excess of intensity above a background thus defined. This intensity of the last visible lines is only a small fraction of the intensity they would have as separate lines. These fractions as determined from the curves are given in Table V.

TABLE V

Residual Intensities of Balmer Lines

$F_0=20$		$F_0=2$	$F_0=1{\cdot}4$		$F_0=0{\cdot}2$	
$n=$ 9	0·92	$n=16$	0·58	0·86	$n=26$	0·51
10	0·63	17	0·34	0·63	27	0·33
11	0·34	18	0·16	0·40	28	0·16
12	0·02	19	0·05	0·20	29	0·09
		20	0·01	0·07	30	0·03
		21		0·01	31	0·01

These residual fractions, as was to be expected, appear to depend on $F_0 n^5$; so for every value of F_0 we may derive the residual intensity for each of the lines. The disappearance of the higher members of the Balmer series is represented in fig. 7. In each of its parts the lower and left-hand curve indicates what fraction of the intensity of each line is dissolved into continuous radiation from which the residual upper part of the line rises as line radiation. A large part of this line radiation, in consequence of its broadening, merges into a continuous background, and only the part above the upper and right-hand curve will be really visible as a separate line. The part of the line intensity which theoretically remains line radiation, but practically is smoothed to continuous background, is broken up into dots in our fig. 7. We see that the disappearance of the higher members of the Balmer series is chiefly determined by the merging of adjacent lines ; the dissolution of the orbits by electric fields plays only a secondary rôle.

These results hold strictly for a gas of constant density and temperature, characterized by a certain value of F_0. We may apply them, however, to such celestial objects where we observe a spectrum of emission lines, *e.g.* in the case of extensive atmospheres, gaseous nebulæ and chromospheres. By observing the number of visible lines and the decrease of their intensity with n we may find a mean value of F_0 and so derive data about density and electron pressure in these objects.

This method was applied to the data given by Menzel and Cillié for the chromosphere.* Owing to lack of sufficient resolving power of the instrument and on account of blending with metal lines the number of visible hydrogen lines could not be exactly ascertained ; accordingly, the measured intensities of the higher members of the Balmer series have been

* *Harvard Circular* No. 410 ; *Astroph. Journal*, 85, 88.

used, as given *loc. cit.*, Table I, pp. 90–91. Their log E was compared with what they should be after simple theory. Since the stimulated transitions up and down between the highest levels, the ionized state included, are more important than the spontaneous transitions (cf. *M.N.*, 96, 788), the partition between these levels will probably be nearer to the thermodynamical equilibrium than to the case of a pure capture spectrum. Hence in this first approximation a normal decrease $\sim n^{-1}$ was assumed. In this way the line number was derived where the residual intensity was 50 per cent. For the three columns of data we found $n=30$ for film 49*a*, $n=26$ for 48*a*, $n=28$ for 49*b*. From the values of Table V this corresponds to $F_0=0{\cdot}10$, $0{\cdot}20$, $0{\cdot}14$. Assuming $F_0=0{\cdot}15$, we find from the expression $F_0=2{\cdot}61N^{2/3}e$ that $\log N=12{\cdot}10$. This is the total number of charged particles, ions and electrons together per c.c. Taking their numbers equal, we have $\log N_e=\log N_i=11{\cdot}80$. Assuming a temperature of 5000°, we find the combined pressure $2P=0{\cdot}9$ dynes/cm.^{2}.

From the intensity of the Balmer continuum, making use of the theoretical value of the absorption coefficient, Menzel and Cillié found $\log N_iN_e=23{\cdot}18$ (*Harv. Circ.* No. 410, p. 23), hence, in the case of a pure hydrogen chromosphere $\log N_i=\log N_e=11{\cdot}59$ (for $T=10{,}000°$; for 6000°, $\log N=11{\cdot}42$). The results are accordant as to the order of magnitude. An exact correspondence cannot be expected, first because Menzel and Cillié's result holds for the base of the chromosphere, whereas our value represents average conditions. Moreover, in the emission of the Balmer continuum only the interaction of hydrogen ions with the electrons plays a rôle, whereas the Stark broadening depends on all the charged particles, which are probably produced in large measure by the ionization of metal atoms.

The problem of deriving the behaviour of the Balmer series as absorption lines in ordinary stellar spectra, by combining our results for the different layers of an atmosphere, is left for further researches.

Summary.—The theoretical work of Robertson and Dewey, and of Lanczos on the dissolution of the highest levels of an atom by electric fields is applied to the disappearance of the higher members of the Balmer series. Their intensity curves in the emission spectrum of a partly ionized gas are computed. The merging of the highest Balmer lines into continuous radiation allows the derivation of the electron density in an atmosphere by means of the number of visible lines. This is applied to the solar chromosphere.

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Universiteit van Amsterdam.

ON A GENERALIZATION OF LINDBLAD'S THEORY OF STAR-STREAMING.

S. Chandrasekhar.

1. *Introduction.*—The dynamics of a stellar system can be studied by one of two methods. The first method consists in specifying in some detail the character of the orbits described by the individual stars and then seeking for a distribution function f of the space and the momenta co-ordinates which are consistent with the derived motions and the equation of continuity. In the second method, on the other hand, no attempt is made to describe the trajectories of the individual stars, and the equations of motion are discussed only to obtain the first integrals consistent with the general large scale symmetry character (*e.g.* spherical, cylindrical, etc.) of the system; the distribution function then follows from an appeal to Jeans's theorem. As an example of an application of these two methods we may refer to the manner in which the dynamics of the globular clusters has been studied. In Eddington's * investigation the first method was adopted. However, as Shiveshwarkar † has shown, results identical with Eddington's can be obtained by the second method. In this particular instance Shiveshwarkar's solution of the dynamics of the globular clusters is mathematically the more elegant. But Eddington's treatment gives greater insight into the actual state of motions in the globular cluster—in particular, it makes the dependence of the ratio of the axes of the velocity ellipsoid on the distance ‡ easily understood. When we come to treat the dynamics of the galaxy the situation is not so simple. As a model of the galaxy we may consider a stellar system with cylindrical symmetry. But an application of Jeans's theorem shows that for such a system radial star-streaming is not possible. This is in direct disagreement with the results of observation. At the same time, as Oort § and also Shiveshwarkar ‖ have shown, the dynamical properties of stellar systems with cylindrical symmetry have certain striking resemblances with the state of motions in the galaxy. In particular, it can be shown¶ that

$$\frac{\overline{|\Theta - \Theta_0|}}{|\bar{\Pi}|} = \sqrt{1 + \frac{A}{\omega_0}}, \tag{1}$$

where Θ_0 and ω_0 are the linear rotational and angular velocities respectively, and $\bar{\Pi}$ the mean speed in the radial ϖ direction and A the Oort

* A. S. Eddington, *M.N.*, **74**, 5, 1913; **75**, 366, 1914.
† S. W. Shiveshwarkar, *M.N.*, **96**, 749, 1936.
‡ The relation in question is: $\bar{R} : \bar{\Theta} : \bar{\Phi} = (1 + kr^2)^{\frac{1}{2}} : 1 : 1$.
§ J. H. Oort, *B.A.N.*, **4**, 269, 1928.
‖ S. W. Shiveshwarkar, *loc. cit.* ¶ Shiveshwarkar, *loc. cit.*, p. 757.

constant. The quantity on the left-hand side is the ratio of the axes of the velocity spheroid in the Θ and the ϖ directions. If the observationally known values of A and ω_0 are substituted in (1) we get for the ratio of the axes the value 0·66 in close agreement with observation. It should, however, be remembered that in the theoretical framework on which (1) is derived the Z-axis of the velocity spheroid is equal to that in the ϖ direction, *i.e.* there is star-streaming in the meridian plane and not in one specified direction. For this reason it is not clear at this stage as to how much significance we can attach to the "correct" prediction which (1) makes for the ratio of the axes parallel to the galactic plane. In view of this difficulty in obtaining a complete dynamical theory of stellar motions in the galaxy at large, it is of value to consider, in the first instance, only the dynamics of stellar motions in our own local neighbourhood. An approach in this direction is more easily possible, as we can now be guided by our knowledge of the kinematical description of the stellar motions in our neighbourhood. This is the method of Lindblad.* Lindblad, however, considered only the motions in the galactic plane, and as a result failed to remove explicitly the fundamental difficulty we have referred to.†

In this paper we shall develop a three-dimensional generalization of Lindblad's theory. This generalization removes the difficulty with regard to the ratio of the axes. This could probably have been anticipated, but the analysis discloses certain new features concerning the "stability" of motions which are of some interest.

Our object then is to obtain, in the first instance, a dynamical interpretation of the state of stellar motions in the neighbourhood (*i.e.* within 500 parsecs) of the Sun. This implies that we have adequate information concerning the character of the motions for which we are seeking a dynamical theory. We shall begin therefore by a brief kinematical description of the stellar motions in our neighbourhood.

2. *The State of Stellar Motions in our Neighbourhood.*—To describe the state of motions in our neighbourhood we need to specify first an appropriate standard of rest, *i.e.* solve for the solar motion for a suitably chosen set of stellar objects. There are two standards of rest which are significant. The first is the standard of rest defined by the nearby stars, or the *local standard of rest*. The second is the standard of rest defined by the galaxy as a whole, or the *fundamental standard of rest*.‡ The local standard of rest

* B. Lindblad, *Arkiv för Matem., Astronomi och Fysik*, 20 A, No. 17, 1927. Also *Stockholms Observatoriums Annaler*, **12**, No. 4, 1936.

† For instance, Eddington remarks : "Oort and Lindblad have obtained certain theoretical relations between the intensity of star-streaming (the prolateness of the Schwarzschild velocity ellipsoid) and the magnitude of the differential rotation, and found a rather impressive agreement with observation ; but it may be urged in criticism that they have chosen one out of a number of incompatible conditions for a steady state, and it is not at all clear why this (rather than the rejected conditions) should be retained in an appropriate compromise" (*Halley Lecture*, p. 16).

‡ It will be noticed that these two standards of rest correspond to the following ideal procedure : Let $(S.R)_r$ refer to the standard of rest obtained by considering

can be specified by solving for the solar motion, using the proper motions or the radial velocities, or both, of the nearby stars. The fundamental standard of rest can be similarly obtained by solving for the solar motion, including in the analysis *all* the stellar objects in the galaxy.*

Now, the analysis of the proper motion data (by the method of drift curves) has shown that with respect to the local standard of rest the motions of the stars have a *random* character, *i.e.* in any given direction the number of stars with positive velocities equals the number with negative velocities. On the other hand, there is a definite direction in which there is *preferential motion*, *i.e.* the mean *speed* in this direction is a maximum. This direction of preferential motion (which defines the vertex of star-streaming) is an axis of *symmetry*. Further, the direction of star-streaming is found to point approximately to the galactic centre. Again, with respect to the fundamental standard of rest, the motion of the centroid of the local stars is approximately circular and in a plane at right angles to the axis of cylindrical symmetry of the galaxy. Further, the rotational circular velocity is numerically much larger than the dispersion of the velocities with respect to the local standard of rest.

From the above description of the state of motions in our neighbourhood we can conclude that the *orbits of the individual stars must be approximately circular*. In Lindblad's theory, and in its generalization which we shall consider, it is precisely orbits of this character which are studied.

3. *The Equations of Motion.*—Let us consider the motion of a star in a potential field which has cylindrical symmetry. If ϖ, θ and z are the cylindrical co-ordinates and if V is the potential per unit mass, then V is a function of ϖ and z only. Since we are interested primarily in orbits which are nearly circular, it is convenient to refer the motion of a star to an appropriately chosen rotating frame of reference.

Let us first consider the equations of motion in a rotating frame of reference which has its origin at a constant distance ϖ_1 from the origin of the (ϖ, θ, z) frame of reference and which further rotates about the axis of symmetry with a constant angular velocity ω_1. We choose the axes (ξ, η, ζ) of the rotating frame of reference in such a manner that the ξ-axis points to the axis of symmetry, the η-axis is in the transverse direction and the ζ-axis is parallel to the z-axis. The kinetic energy T per unit mass is given by

$$T = \tfrac{1}{2}(\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2) - \omega_1 \dot{\xi}\eta + \omega_1 \dot{\eta}(\varpi_1 + \xi) + \tfrac{1}{2}\omega_1^2\{(\varpi_1 + \xi)^2 + \eta^2\}. \qquad (2)$$

---

stars in a volume of space of radius r about the region considered. Then $\lim_{r\to 0} (S.R)_r$ is the local standard of rest, while $\lim_{r\to\infty} (S.R)_r$ is the fundamental standard of rest.

* This method cannot be adopted in practice for obvious reasons. Actually the fundamental standard of rest is obtained by solving for the solar motion from the radial velocities of the globular clusters. It should be pointed out that an argument of *plausibility* is involved here, *i.e.* we *assume* from the relation of symmetry which the system of the globular clusters bears to the galaxy as a whole that the standard of rest defined by these objects will approximately be the same as the fundamental standard of rest.

The generalized components of momenta (p_ξ, p_η, p_ζ) are given by

$$\left.\begin{aligned} p_\xi &= \frac{\partial T}{\partial \dot{\xi}} = \dot{\xi} - \omega_1 \eta, \\ p_\eta &= \frac{\partial T}{\partial \dot{\eta}} = \dot{\eta} + \omega_1(\varpi_1 + \xi), \\ p_\zeta &= \frac{\partial T}{\partial \dot{\zeta}} = \dot{\zeta}. \end{aligned}\right\} \tag{3}$$

The Lagrangian function L and the Hamiltonian H are given by

$$L = T - V = \tfrac{1}{2}(p_\xi^2 + p_\eta^2 + p_\zeta^2) - V(\varpi, z), \tag{4}$$

$$\begin{aligned} H &= \sum_{s=\xi,\eta,\zeta} \dot{q}_s p_s - L \\ &= \tfrac{1}{2}(p_\xi^2 + p_\eta^2 + p_\zeta^2) + V + \omega_1\{\eta p_\xi - (\varpi_1 + \xi)p_\eta\}. \end{aligned} \tag{5}$$

The generalized components of force in the ξ, η, ζ directions are respectively

$$-\frac{\varpi_1 + \xi}{\varpi}\frac{\partial V}{\partial \varpi}; \qquad -\frac{\eta}{\varpi}\frac{\partial V}{\partial \varpi}; \qquad -\frac{\partial V}{\partial \zeta}. \tag{6}$$

The Lagrangian equations of motion are

$$\ddot{\xi} - 2\omega_1\dot{\eta} - \omega_1^2(\varpi_1 + \xi) = -\frac{\varpi_1 + \xi}{\varpi}\frac{\partial V}{\partial \varpi}, \tag{7}$$

$$\ddot{\eta} + 2\omega_1\dot{\xi} - \omega_1^2\eta = -\frac{\eta}{\varpi}\frac{\partial V}{\partial \varpi}, \tag{8}$$

$$\ddot{\zeta} = -\frac{\partial V}{\partial \zeta}. \tag{9}$$

The angular momentum integral is obtained by taking the difference between the integral $H = \text{constant}$ and the energy integral $T + V = \text{constant}$. We therefore have, *cf.* equation (5),

$$\eta p_\xi - (\varpi_1 + \xi)p_\eta = \text{constant}, \tag{10}$$

or by (3)

$$\eta\dot{\xi} - (\varpi_1 + \xi)\dot{\eta} - \omega_1\{(\varpi_1 + \xi)^2 + \eta^2\} = \text{constant}. \tag{11}$$

Equation (11) is the angular momentum integral. So far the equations are quite general. We shall now assume that the rotating frame of reference has been so chosen that ξ, η and ζ are small compared with ϖ and ϖ_1 and we neglect all quantities of the second order. To this order of accuracy

$$\varpi = \varpi_1 + \xi, \tag{12}$$

and hence

$$\left(\frac{\partial V}{\partial \varpi}\right)_{\varpi,\zeta} = \left(\frac{\partial V}{\partial \varpi}\right)_1 + \xi\left(\frac{\partial^2 V}{\partial \varpi^2}\right)_1 + \zeta\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1, \tag{13}$$

$$\left(\frac{\partial V}{\partial \zeta}\right)_{\varpi,\zeta} = \left(\frac{\partial V}{\partial \zeta}\right)_1 + \xi\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1 + \zeta\left(\frac{\partial^2 V}{\partial \zeta^2}\right)_1. \tag{14}$$

In (13) and (14) the suffix 1 indicates that the value of the quantity inside the brackets is to be evaluated at the origin of the (ξ, η, ζ) system. The equations of motion (7), (8) and (9) now simplify to the forms

$$\ddot{\xi} - 2\omega_1\dot{\eta} - \omega_1^2(\varpi_1 + \xi) = -\left\{\left(\frac{\partial V}{\partial \varpi}\right)_1 + \xi\left(\frac{\partial^2 V}{\partial \varpi^2}\right)_1 + \zeta\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1\right\}, \tag{15}$$

$$\ddot{\eta} + 2\omega_1\dot{\xi} - \omega_1^2\eta = -\frac{\eta}{\varpi_1}\left(\frac{\partial V}{\partial \varpi}\right)_1, \tag{16}$$

$$\ddot{\zeta} = -\left(\frac{\partial V}{\partial \zeta}\right)_1 - \xi\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1 - \zeta\left(\frac{\partial^2 V}{\partial \zeta^2}\right)_1. \tag{17}$$

Now, the orbits of the individual stars will be labelled by the values of the angular momentum integral. Since we are going to consider only nearly circular orbits, it is convenient to choose for comparison a kinematically possible circular path which has the same angular momentum about the axis of symmetry as the actual orbit. Hence for the (ξ, η, ζ) system chosen in this manner,

$$\eta\dot{\xi} - (\varpi_1 + \xi)\dot{\eta} - \omega_1\{(\varpi_1 + \xi)^2 + \eta^2\} = \omega_1\varpi_1^2, \tag{18}$$

or to the order of accuracy we are working

$$\varpi_1\dot{\eta} + 2\omega_1\varpi_1\xi = 0. \tag{19}$$

Further, we can arrange that

$$\omega_1^2\varpi_1 = \left(\frac{\partial V}{\partial \varpi}\right)_1. \tag{20}$$

(It should be emphasized at this stage that we are comparing the actual orbit described by the star with a circular path which need not be a dynamically possible orbit.) The equations of motion now reduce to

$$\ddot{\xi} - 2\omega_1\dot{\eta} - \xi\left\{\omega_1^2 - \left(\frac{\partial^2 V}{\partial \varpi^2}\right)_1\right\} + \zeta\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1 = 0, \tag{21}$$

$$\ddot{\eta} + 2\omega_1\dot{\xi} = 0, \tag{22}$$

$$\ddot{\zeta} + \xi\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1 + \zeta\left(\frac{\partial^2 V}{\partial \zeta^2}\right)_1 = -\left(\frac{\partial V}{\partial \zeta}\right)_1. \tag{23}$$

From (19) and (22) we have

$$\dot{\eta} = -2\omega_1\dot{\xi}. \tag{24}$$

Substituting (24) for the term in $\dot{\eta}$ in (21) we have

$$\ddot{\xi} + 4\omega_1^2\xi - \left\{\omega_1^2 - \left(\frac{\partial^2 V}{\partial \varpi^2}\right)_1\right\}\xi + \left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_1 \zeta = 0. \tag{25}$$

Defining the Oort constant A_1 at $(\varpi_1, 0)$ by

$$A_1 = -\frac{1}{4\omega_1}\left\{\omega_1^2 - \left(\frac{\partial^2 V}{\partial \varpi^2}\right)_1\right\}, \tag{26}$$

(25) reduces to

$$\ddot{\xi}+4\omega_1(\omega_1+A_1)\xi+\left(\frac{\partial^2 V}{\partial\varpi\partial\zeta}\right)_1\zeta=0. \tag{27}$$

Equations (23) and (27) can now be written in the forms

$$\ddot{\xi}+\alpha\xi+\beta\zeta=0, \tag{28}$$

$$\ddot{\zeta}+\beta\xi+\gamma\zeta=-Q, \tag{29}$$

where

$$\alpha=4\omega_1(\omega_1+A_1)\,;\qquad \beta=\left(\frac{\partial^2 V}{\partial\varpi\partial\zeta}\right)_1\,;\qquad \gamma=\left(\frac{\partial^2 V}{\partial\zeta^2}\right)_1, \tag{30}$$

and

$$Q=\left(\frac{\partial V}{\partial\zeta}\right)_1. \tag{31}$$

4. *Solution of the Equations of Motion.*—Equations (28) and (29) are two simultaneous linear differential equations for ξ and ζ. They can be solved by standard methods and we easily find that

$$\xi=X_1\sin g_1(t-\delta_1)+X_2\sin g_2(t-\delta_2)+\frac{\beta}{\alpha\gamma-\beta^2}Q, \tag{32}$$

$$\zeta=\frac{g_1^2-\alpha}{\beta}X_1\sin g_1(t-\delta_1)+\frac{g_2^2-\alpha}{\beta}X_2\sin g_2(t-\delta_2)-\frac{\alpha}{\alpha\gamma-\beta^2}Q, \tag{33}$$

where X_1, X_2, δ_1 and δ_2 are constants of integration and g_1^2 and g_2^2 are the characteristic roots of the matrix

$$\begin{vmatrix} \alpha & \beta \\ \beta & \gamma \end{vmatrix}, \tag{34}$$

i.e. g_1^2 and g_2^2 satisfy the equation

$$(\alpha-g^2)(\gamma-g^2)-\beta^2=0. \tag{35}$$

From (35) we derive that

$$g_1^2=\tfrac{1}{2}\{(\alpha+\gamma)+\sqrt{(\alpha-\gamma)^2+4\beta^2}\}, \tag{36}$$

$$g_2^2=\tfrac{1}{2}\{(\alpha+\gamma)-\sqrt{(\alpha-\gamma)^2+4\beta^2}\}. \tag{37}$$

The solutions (32) and (33) are valid only if g_1 and g_2 are both real. From (36) it follows that g_1^2 is always positive; on the other hand, g_2^2 becomes negative when

$$(\alpha+\gamma)^2<(\alpha-\gamma)^2+4\beta^2, \tag{38}$$

or

$$\alpha\gamma<\beta^2. \tag{39}$$

If g_2^2 is negative, then ξ and ζ will involve terms in exp. $\{\pm\sqrt{-g_2^2}t\}$. In other words, if (39) is true, ξ and ζ will increase exponentially with time and the motions of the character considered will be "*unstable*." The discussion of the meaning of this instability is postponed to §§ 10 and 12, but

meantime we shall assume that g_2 is real and that the solutions (32) and (33) are valid.

From (32) and (33) we derive

$$\dot{\xi} = g_1 X_1 \cos g_1(t-\delta_1) + g_2 X_2 \cos g_2(t-\delta_2), \tag{40}$$

$$\dot{\zeta} = g_1 \frac{g_1^2-\alpha}{\beta} X_1 \cos g_1(t-\delta_1) + g_2 \frac{g_2^2-\alpha}{\beta} X_2 \cos g_2(t-\delta_2). \tag{41}$$

From (24) and (32) we have

$$\dot{\eta} = -2\omega_1 X_1 \sin g_1(t-\delta_1) - 2\omega_1 X_2 \sin g_2(t-\delta_2) - \frac{2\omega_1\beta}{\alpha\gamma-\beta^2} Q, \tag{42}$$

or integrating we find that

$$\eta = \frac{2\omega_1}{g_1} X_1 \cos g_1(t-\delta_1) + \frac{2\omega_1}{g_2} X_2 \cos g_2(t-\delta_2) - \frac{2\omega_1\beta}{\alpha\gamma-\beta^2} Q(t-\delta_3), \tag{43}$$

where δ_3 is another constant of integration. We have thus solved the dynamical problem.

5. *The Integrals $\mathcal{J}_1$ and $\mathcal{J}_2$.*—We can rewrite equations (33) and (42) in the forms

$$\beta\zeta + \frac{\alpha\beta}{\alpha\gamma-\beta^2} Q = (g_1^2-\alpha) X_1 \sin g_1(t-\delta_1) + (g_2^2-\alpha) X_2 \sin g_2(t-\delta_2), \tag{44}$$

$$\frac{\dot{\eta}}{2\omega_1} + \frac{\beta}{\alpha\gamma-\beta^2} Q = -X_1 \sin g_1(t-\delta_1) - X_2 \sin g_2(t-\delta_2). \tag{45}$$

From (44) and (45) we derive

$$\frac{1}{g_1^2-g_2^2}\left\{(g_2^2-\alpha)\left[\frac{\dot{\eta}}{2\omega_1} + \frac{\beta}{\alpha\gamma-\beta^2} Q\right] + \beta\left[\zeta + \frac{\alpha}{\alpha\gamma-\beta^2} Q\right]\right\} = X_1 \sin g_1(t-\delta_1), \tag{46}$$

$$\frac{1}{g_1^2-g_2^2}\left\{(g_1^2-\alpha)\left[\frac{\dot{\eta}}{2\omega_1} + \frac{\beta}{\alpha\gamma-\beta^2} Q\right] + \beta\left[\zeta + \frac{\alpha}{\alpha\gamma-\beta^2} Q\right]\right\} = -X_2 \sin g_2(t-\delta_2). \tag{47}$$

Similarly from (40) and (41) we derive

$$\frac{1}{g_1(g_1^2-g_2^2)}\{(g_2^2-\alpha)\dot{\xi} - \beta\dot{\zeta}\} = -X_1 \cos g_1(t-\delta_1), \tag{48}$$

$$\frac{1}{g_2(g_1^2-g_2^2)}\{(g_1^2-\alpha)\dot{\xi} - \beta\dot{\zeta}\} = X_2 \cos g_2(t-\delta_2). \tag{49}$$

From (46), (47), (48) and (49) we find that

$$\left\{\left(\frac{\dot{\eta}}{2\omega_1} + \frac{\beta}{\alpha\gamma-\beta^2} Q\right) + \frac{\beta}{g_2^2-\alpha}\left(\zeta + \frac{\alpha}{\alpha\gamma-\beta^2} Q\right)\right\}^2 + \frac{1}{g_1^2}\left(\dot{\xi} - \frac{\beta}{g_2^2-\alpha}\dot{\zeta}\right)^2 = \frac{(g_1^2-g_2^2)^2}{(g_2^2-\alpha)^2} X_1^2 = \mathcal{J}_1, \tag{50}$$

$$\left\{\frac{g_1^2-\alpha}{\beta}\left(\frac{\dot{\eta}}{2\omega_1} + \frac{\beta}{\alpha\gamma-\beta^2} Q\right) + \left(\zeta + \frac{\alpha}{\alpha\gamma-\beta^2} Q\right)\right\}^2 + \frac{1}{g_2^2}\left(\frac{g_1^2-\alpha}{\beta}\dot{\xi} - \dot{\zeta}\right)^2 = \frac{(g_1^2-g_2^2)^2}{\beta^2} X_2^2 = \mathcal{J}_2. \tag{51}$$

$\mathcal{J}_1$ and $\mathcal{J}_2$ are two first integrals of the equations of motion in the frame of reference considered. The integrals simplify considerably in one special case, namely, when $\beta = 0$. In this case $g_1^2 = \alpha$ and $g_2^2 = \gamma$. Further, from (36) we can show that $(g_1^2 - \alpha)/\beta \to 0$ as $\beta \to 0$. Hence for this special case (50) and (51) reduce to

$$\frac{\dot{\eta}^2}{4\omega_1^2} + \frac{\dot{\xi}^2}{g_1^2} = \mathcal{J}_1, \tag{52}$$

$$\left(\zeta + \frac{1}{\gamma}Q\right)^2 + \frac{1}{\gamma}\dot{\zeta}^2 = \mathcal{J}_2. \tag{53}$$

6. *Transformation to the Local Standard of Rest.*—In the method that was adopted in §§ 4 and 5 for solving the dynamical problem the choice of the particular (ξ, η, ζ) system was dependent on the value of the angular momentum integral characterizing the particular orbit considered. Hence, stars with different values of the angular momentum integral will be referred to in different frames of references. On the other hand, we must refer the motion of all the nearby stars to the same frame of reference in order to study their motions with respect to the local standard of rest. For this purpose we consider a (ξ_0, η_0, ζ_0) system which rotates about the axis of symmetry with a constant angular velocity ω_0 and the origin of which is at a constant distance ϖ_0 from the origin of the fundamental standard of rest. Further, we consider stars which all pass through a small volume near the origin of the (ξ_0, η_0, ζ_0) system. Further, we assume that

$$\omega_0^2 \varpi_0 = \left(\frac{\partial V}{\partial \varpi}\right)_0. \tag{54}$$

The transformations required to pass from the (ξ, η, ζ) system to the (ξ_0, η_0, ζ_0) system are to the first order (*cf.* Lindblad, *loc. cit.*),

$$\dot{\xi}_0 = \dot{\xi}\,; \qquad \dot{\eta}_0 = \dot{\eta}\left(1 + \frac{A_1}{\omega_1}\right); \qquad \dot{\zeta}_0 = \dot{\zeta}. \tag{55}$$

To obtain the integrals in the (ξ_0, η_0, ζ_0) system we have to substitute for $\dot{\xi}$, $\dot{\eta}$ and $\dot{\zeta}$ according to (55) in equations (50) and (51). Further, to the order of accuracy we are working, we can replace A_1 and ω_1 by the values A and ω they take at the origin of the (ξ_0, η_0, ζ_0) system. The integrals in the (ξ_0, η_0, ζ_0) system are therefore given by

$$\left\{\left(\frac{\dot{\eta}_0}{2(A+\omega)} + \frac{\beta}{\alpha\gamma - \beta^2}Q\right) + \frac{\beta}{g_2^2 - \alpha}\left(\zeta_0 + \frac{\alpha}{\alpha\gamma - \beta^2}Q\right)\right\}^2 + \frac{1}{g_1^2}\left(\dot{\xi}_0 - \frac{\beta}{g_2^2 - \alpha}\dot{\zeta}_0\right)^2 = I_1, \tag{56}$$

$$\left\{\frac{g_1^2 - \alpha}{\beta}\left(\frac{\dot{\eta}_0}{2(A+\omega)} + \frac{\beta}{\alpha\gamma - \beta^2}Q\right) + \left(\zeta_0 + \frac{\alpha}{\alpha\gamma - \beta^2}Q\right)\right\}^2 + \frac{1}{g_2^2}\left(\frac{g_1^2 - \alpha}{\beta}\dot{\xi}_0 - \dot{\zeta}_0\right)^2 = I_2. \tag{57}$$

In (56) and (57) α, β and γ are also to be evaluated at the origin of the (ξ_0, η_0, ζ_0) system.

7. *The Distribution Function characterizing the Local Motions.*—In the (ξ_0, η_0, ζ_0) system the equations of motion are in Hamiltonian form and we can therefore apply all the general theorems valid for a conservative dynamical system, in particular Liouville's theorem. Thus, if f is the distribution function which specifies the number of stars which have the space co-ordinates in the range

$$(\xi_0, \xi_0 + d\xi_0\,; \quad \eta_0, \eta_0 + d\eta_0\,; \quad \zeta_0, \zeta_0 + d\zeta_0)$$

and the momenta co-ordinates in the range

$$(p_{\xi_0}, p_{\xi_0} + dp_{\xi_0}\,; \quad p_{\eta_0}, p_{\eta_0} + dp_{\eta_0}\,; \quad p_{\zeta_0}, p_{\zeta_0} + dp_{\zeta_0}),$$

then

$$\frac{\partial f}{\partial t} + [H, f] = 0, \tag{58}$$

where $[H, f]$ is the classical Poisson bracket expression. Equation (58) is now regarded as a partial differential equation for f. It can be solved by writing down the appropriate subsidiary equations. In this case the subsidiary equations are the Hamiltonian equations of motion, and hence f is an arbitrary function of the first integrals of the equations of motion. For the dynamical system studied in the previous sections we should have

$$f = f(I_1, I_2), \tag{59}$$

where I_1 and I_2 are given by (56) and (57). Hence

$$dN = f(I_1, I_2) d\xi_0 d\eta_0 d\zeta_0 dp_{\xi_0} dp_{\eta_0} dp_{\zeta_0}. \tag{60}$$

From (3) we derive that

$$dp_{\xi_0} dp_{\eta_0} dp_{\zeta_0} = d\dot{\xi}_0 d\dot{\eta}_0 d\dot{\zeta}_0. \tag{61}$$

Hence the number of stars in unit volume in the velocity range

$$(\dot{\xi}_0, \dot{\xi}_0 + d\dot{\xi}_0\,; \quad \dot{\eta}_0, \dot{\eta}_0 + d\dot{\eta}_0\,; \quad \dot{\zeta}_0, \dot{\zeta}_0 + d\dot{\zeta}_0)$$

is given by

$$f(I_1, I_2) d\dot{\xi}_0 d\dot{\eta}_0 d\dot{\zeta}_0. \tag{62}$$

If we accept for the distribution function a generalized Schwarzschild ellipsoidal type, then we have to choose for f the form

$$f = f(I_1 + \lambda^2 I_2), \tag{63}$$

where λ is an arbitrary constant. Finally, if we assume for f a "Gaussian curve," then

$$f = \text{constant } e^{-k(I_1 + \lambda^2 I_2)}. \tag{64}$$

This solves the formal problem.

8. *Characteristics of the Predicted Motions near the Galactic Plane.*—Before we proceed to a discussion of the general distribution function (63) we shall first consider the special case of the motions near the galactic

plane. If the galactic plane is a plane of symmetry and if the local centroid is situated on the galactic plane, then we should have

$$Q=-\left(\frac{\partial V}{\partial \zeta}\right)_0=0\,; \qquad \beta=\left(\frac{\partial^2 V}{\partial \varpi \partial \zeta}\right)_0=0. \tag{65}$$

For this case the integrals reduce to

$$I_1=\frac{\dot\eta_0{}^2}{4(A+\omega)^2}+\frac{\dot\xi_0{}^2}{g_1{}^2}, \tag{66}$$

$$I_2=\zeta_0{}^2+\frac{\dot\zeta_0{}^2}{g_2{}^2}. \tag{67}$$

Hence by (63) we have in this case

$$f=f\left\{\frac{\dot\eta_0{}^2}{4(A+\omega)^2}+\frac{\dot\xi_0{}^2}{g_1{}^2}+\lambda^2\left(\zeta_0{}^2+\frac{\dot\zeta_0{}^2}{g_2{}^2}\right)\right\}, \tag{68}$$

where, further,

$$g_1{}^2=\alpha=4\omega(\omega+A), \tag{69}$$

$$g_2{}^2=\gamma=\left(\frac{\partial^2 V}{\partial \zeta^2}\right)_0. \tag{70}$$

From (68) it follows that

$$\overline{|\dot\xi_0|}:\overline{|\dot\eta_0|}:\overline{|\dot\zeta_0|}=g_1:2(A+\omega):g_2/\lambda, \tag{71}$$

or by (69) and (70)

$$\overline{|\dot\xi_0|}:\overline{|\dot\eta_0|}:\overline{|\dot\zeta_0|}=1:\sqrt{1+\frac{A}{\omega}}:\frac{1}{2\lambda}\sqrt{\frac{\gamma}{\omega(\omega+A)}}. \tag{72}$$

The ratio of the ξ- to the η-axis of the velocity ellipsoid is the same as in the Lindblad theory. But (72) does not require the ζ-axis to be the same as the ξ-axis. We can in fact choose λ^2 in such a way that the η- and the ζ-axes are equal.

At this stage in the development of the theory we cannot attribute any deep meaning to the observed fact concerning the equality of the η- and the ζ-axis. However, it is seen that a generalization of Lindblad's theory removes the fundamental difficulty which has been experienced in providing a satisfactory theory for star-streaming.

9. *Characteristics of the Predicted Motions away from the Galactic Plane.* —In the general case we can write

$$\begin{aligned}
I_1+\lambda^2 I_2=&\left\{\left(\frac{\dot\eta_0}{2(A+\omega)}+\frac{\beta}{\alpha\gamma-\beta^2}Q\right)+\frac{\beta}{g_2{}^2-\alpha}\left(\zeta_0+\frac{\alpha}{\alpha\gamma-\beta^2}Q\right)\right\}^2\\
&+\lambda^2\left\{\frac{g_1{}^2-\alpha}{\beta}\left(\frac{\dot\eta_0}{2(A+\omega)}+\frac{\beta}{\alpha\gamma-\beta^2}Q\right)+\left(\zeta_0+\frac{\alpha}{\alpha\gamma-\beta^2}Q\right)\right\}^2\\
&+\dot\xi_0{}^2\left\{\frac{1}{g_1{}^2}+\lambda^2\frac{(g_1{}^2-\alpha)^2}{g_2{}^2\beta^2}\right\}\\
&+\dot\zeta_0{}^2\left\{\frac{\beta^2}{g_1{}^2(g_2{}^2-\alpha)^2}+\lambda^2\frac{1}{g_2{}^2}\right\}\\
&-2\dot\xi_0\dot\zeta_0\left\{\frac{\beta}{g_1{}^2(g_2{}^2-\alpha)}+\lambda^2\frac{g_1{}^2-\alpha}{g_2{}^2\beta}\right\}.
\end{aligned} \tag{73}$$

(73) is a quadratic form in $\dot{\xi}_0$, $\dot{\eta}_0$ and $\dot{\zeta}_0$ and can be brought to its normal form by an appropriately chosen linear transformation. It is seen that one of the axes of the velocity ellipsoid is in the transverse η_0 direction, but the other two principal axes are not in the ξ_0 and ζ_0 directions. The terms in $\dot{\xi}_0$ and $\dot{\zeta}_0$ can be brought into the normal form by a rotation of the axes in the (ξ_0, ζ_0) plane by an angle ψ, where

$$\tan 2\psi = \frac{-2\left\{\dfrac{\beta}{g_1^2(g_2^2-\alpha)} + \lambda^2\dfrac{g_1^2-\alpha}{g_2^2\beta}\right\}}{\dfrac{1}{g_1^2}\left\{1-\dfrac{\beta^2}{(g_2^2-\alpha)^2}\right\} + \lambda^2\dfrac{1}{g_2^2}\left\{\dfrac{(g_1^2-\alpha)^2}{\beta^2}-1\right\}}. \tag{74}$$

This expression for the *tilt* of the velocity axes in the (ξ_0, ζ_0) plane depends on the constant λ. In one special case (74) takes an explicit form, namely, when $\beta^2 = \alpha\gamma$, *i.e.* just on the "edge" of stability of the type of motions considered.* If $\beta^2 = \alpha\gamma$, then

$$g_1^2 = \alpha + \gamma, \qquad g_2^2 = 0, \qquad (\beta^2 = \alpha\gamma), \tag{75}$$

and (74) reduces to

$$[\tan 2\psi]_{\beta^2=\alpha\gamma} = \tan 2\psi^* = -\frac{2\beta}{\gamma-\alpha}. \tag{76}$$

We shall see in § 11 that (76) implies that, at a height of about 3500 parsecs above the galactic plane, this tilt of the principal axis of the velocity ellipsoid in the meridian plane amounts to as much as 55°.

10. *The Stability of Circular Motions.*—So far we have assumed that the two roots of the matrix

$$\begin{vmatrix} \alpha & \beta \\ \beta & \gamma \end{vmatrix} \tag{77}$$

are positive. But, as we have seen, this is true only if $\beta^2 < \alpha\gamma$. If $\beta^2 > \alpha\gamma$, then the solutions take the forms

$$\xi = X_1 \sin g_1(t-\delta_1) + Y_1 e^{+\sqrt{-g_2^2}t} + Y_2 e^{-\sqrt{-g_2^2}t} + \frac{\beta}{\alpha\gamma-\beta^2}Q, \tag{78}$$

$$\zeta = \frac{g_1^2-\alpha}{\beta}X_1 \sin g_1(t-\delta_1) + \frac{g_2^2-\alpha}{\beta}\left(Y_1 e^{+\sqrt{-g_2^2}t} + Y_2 e^{-\sqrt{-g_2^2}t}\right) - \frac{\alpha}{\alpha\gamma-\beta^2}Q. \tag{79}$$

This means that ξ and ζ tend to infinity exponentially with time. Thus ξ and ζ will cease to be "small" after some time. Hence, if $\beta^2 > \alpha\gamma$, then the description of the motions with respect to the appropriate local centroid should be different from the kinematical description valid for the neighbourhood of the Sun. Equations (78) and (79) suggest that under these circumstances the dispersion of the velocities with respect to the local standard

* There is one other "trivial" case when (76) gives an explicit value for ψ. This occurs on the galactic plane when $\beta = 0$; in this case $\psi = 0$.

of rest must be quite large. In this sense the theory of galactic rotation ceases to be valid when $\beta^2 > \alpha\gamma$. We shall now investigate this condition a little more closely.

(i) *The Potential due to a Point Mass.*—If the potential is due to a point mass M_0 concentrated towards the galactic centre, then we can write

$$V = -GM_0 \frac{1}{(\varpi^2+z^2)^{1/2}}. \tag{80}$$

From (80) we derive that

$$\left.\begin{aligned} \alpha &= GM_0 \frac{\varpi^2+4z^2}{(\varpi^2+z^2)^{5/2}}, \\ \beta &= -GM_0 \frac{3\varpi z}{(\varpi^2+z^2)^{5/2}}, \\ \gamma &= GM_0 \frac{\varpi^2-2z^2}{(\varpi^2+z^2)^{5/2}}. \end{aligned}\right\} \tag{81}$$

Thus for the case considered, the condition $\beta^2 > \alpha\gamma$ is equivalent to

$$\frac{9\varpi^2 z^2}{(\varpi^2+z^2)^5} > \frac{(\varpi^2+4z^2)(\varpi^2-2z^2)}{(\varpi^2+z^2)^5}. \tag{82}$$

After some reductions we find that (82) can be simplified to

$$8z^2 - \varpi^2 > 0, \tag{83}$$

or

$$z > \sqrt{0{\cdot}125}\,\varpi. \tag{84}$$

Hence, for $z > 0{\cdot}353\varpi$ we have instability of the motions considered. Actually, we may expect that the character of the motions considered ceases to be a good approximation for even smaller values of z. We shall return to this matter in the last section.

(ii) *The Interior Points of a Uniform Spheroidal Distribution of Mass.*—In this case we have

$$V = -\tfrac{1}{2}(\mathfrak{D} - \mathfrak{A}\varpi^2 - \mathfrak{B}z^2), \tag{85}$$

where $\mathfrak{A}$, $\mathfrak{B}$ and $\mathfrak{D}$ are positive constants.*

From (85) we derive that

$$\alpha = 4\mathfrak{A}\,; \qquad \beta = 0\,; \qquad \gamma = \mathfrak{B}. \tag{86}$$

The condition for instability is therefore never satisfied inside a uniform spheroidal (or ellipsoidal) distribution of mass. Hence, the motions according to the kinematical description of the stellar motions in the neighbourhood of the Sun will be stable inside a spheroidal distribution of mass.

(iii) *The Potential due to a Point Mass together with that of a Uniform Spheroidal Distribution of Mass.*—For this case

$$V = -\tfrac{1}{2}(\mathfrak{D} - \mathfrak{A}\varpi^2 - \mathfrak{B}z^2) - GM_0 \frac{1}{(\varpi^2+z^2)^{1/2}}, \tag{87}$$

* See, for example, E. J. Routh, *A Treatise on Analytical Statics*, vol. ii, pp. 106–116.

where (see Routh, *loc. cit.*)

$$\mathfrak{A} = \rho\nu G\,; \qquad \mathfrak{B} = \rho\mu G, \tag{88}$$

ρ being the mean density of the uniform spheroidal distribution of mass M_s, and ν and μ are the constants such that

$$2\nu + \mu = 4\pi, \tag{89}$$

$$2\nu a^2 + \mu c^2 = \frac{4\pi a^2 c}{\sqrt{a^2 - c^2}} \tan^{-1}\sqrt{\frac{a^2}{c^2} - 1}. \tag{90}$$

In (90), a and c are respectively the semi-major and minor axes of the spheroid. From (87) we derive that

$$\left.\begin{aligned} \alpha &= 4\mathfrak{A} + GM_0\frac{\varpi^2 + 4z^2}{(\varpi^2 + z^2)^{5/2}}, \\ \beta &= -GM_0\frac{3\varpi z}{(\varpi^2 + z^2)^{5/2}}, \\ \gamma &= \mathfrak{B} + GM_0\frac{\varpi^2 - 2z^2}{(\varpi^2 + z^2)^{5/2}}. \end{aligned}\right\} \tag{91}$$

Instability of the motions considered sets in when

$$G^2M_0{}^2\frac{9\varpi^2 z^2}{(\varpi^2 + z^2)^5} > \left[4\mathfrak{A} + GM_0\frac{\varpi^2 + 4z^2}{(\varpi^2 + z^2)^{5/2}}\right]\left[\mathfrak{B} + GM_0\frac{\varpi^2 - 2z^2}{(\varpi^2 + z^2)^{5/2}}\right]. \tag{92}$$

Equation (92) can be simplified if we measure linear distances in units of the minor axes of the spheroid. If we adopt this convention, equation (92) can be reduced to the form

$$\frac{8z^2 - \varpi^2}{(\varpi^2 + z^2)^4} > 2\epsilon^2\vartheta(1 - \vartheta) + \epsilon\frac{(2 - \vartheta)\varpi^2 + 4(2\vartheta - 1)z^2}{(\varpi^2 + z^2)^{5/2}}, \tag{93}$$

where

$$\epsilon = 3\frac{M_s}{M_0}\frac{1}{a^2}, \tag{94}$$

$$\vartheta = \frac{a^2}{a^2 - 1} - \frac{a^2}{(a^2 - 1)^{3/2}}\tan^{-1}\sqrt{a^2 - 1}. \tag{95}$$

In (95), a is now the semi-major axis of the spheroidal distribution of mass measured in units of the minor axis. We have

$$\left.\begin{array}{llllllllll} a & 1, & 2, & 3, & 4, & 5, & 6, & 8, & 10, & \infty. \\ \vartheta = \frac{1}{3}, & 0{\cdot}527, & 0{\cdot}635, & 0{\cdot}704, & 0{\cdot}751, & 0{\cdot}785, & 0{\cdot}831, & 0{\cdot}861, & 1. \end{array}\right\} \tag{96}$$

It is readily seen that (93) with an equality sign represents a closed symmetrical curve which passes through the origin where it is tangential to the line (84). The point of intersection z_0 of the curve with the z-axis is obtained by putting $\varpi = 0$ in (93). We then obtain

$$-\frac{8}{z_0{}^6} + \frac{4(2\vartheta - 1)}{z_0{}^3}\epsilon + 2\vartheta(1 - \vartheta)\epsilon^2 = 0, \tag{97}$$

or

$$\left(\vartheta\epsilon - \frac{2}{z_0{}^3}\right)\left((1-\vartheta)\epsilon + \frac{2}{z_0{}^3}\right) = 0. \tag{98}$$

The second term in (98) is positive. Hence

$$z_0{}^3 = \frac{2}{\vartheta\epsilon}, \tag{99}$$

or by (94)

$$z_0 = \left(\frac{2}{3}\frac{a^2}{\vartheta}\frac{M_0}{M_s}\right)^{1/3}. \tag{100}$$

If the right-hand side of (100) is less than unity, then the region of instability is entirely inside the spheroidal distribution of mass. (See figs. 1 and 2.) If the right-hand side of (100) is greater than unity, then the curve separating

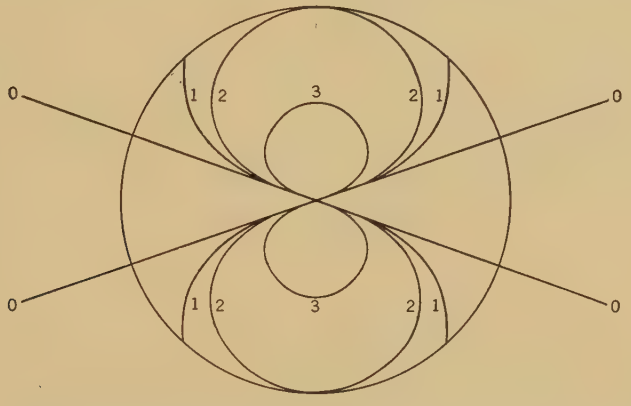


FIG. 1.—*The regions of stability and instability according to equation* (93). *The potential is due to a spherical distribution of mass* M_s *and a point mass* M_0 *at the centre of the sphere. The curves* 1, 2 *and* 3 *correspond to* M_s/M_0, *having the values* 1, 2 *and* 16 *respectively. The lines* (0, 0) *are for a point mass only* (cf. *equation* (83)).

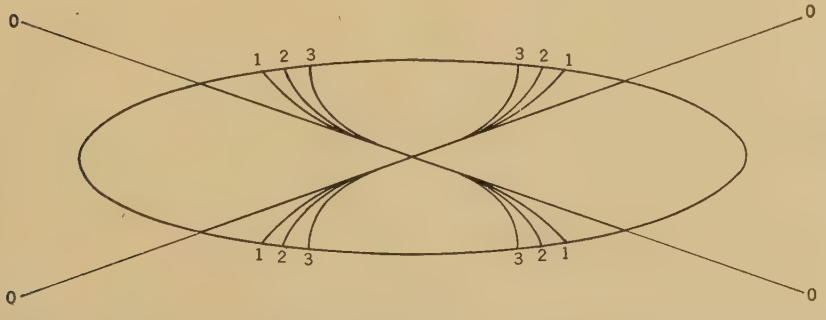


FIG. 2.—*The regions of stability and instability according to equation* (93). *The potential is due to a spheroidal distribution of mass,* ($\vartheta = \frac{2}{3}$, $a = 3{\cdot}41$), M_s *and a point mass* M_0 *at the centre of the spheroid. The curves* 1, 2 *and* 3 *correspond to* M_s/M_0, *having the values* 0·5, 1 *and* 2 *respectively. The lines* (0, 0) *are for a point mass only* (cf. *equation* (83)).

the regions of stability and instability according to (93) cuts the ellipse

$$z^2+\frac{\varpi^2}{a^2}=1. \tag{101}$$

The continuation of the curve beyond the point of intersection has no physical meaning.

In figs. 1 and 2 the regions of stability inside (i) a spherical and (ii) a spheroidal ($a=3\cdot41$, $\vartheta=\frac{2}{3}$) distribution of mass are shown for different values of the ratio M_s/M_0. The figures illustrate a general principle that the "edge" of stability depends very largely on the potential due to the point mass unless M_s/M_0 is very great. However, there are observational indications that $M_s/M_0 \sim 1/2$. Hence, we can say that (84) will give approximately the heights to which we can go in the z-co-ordinate for a given value of ϖ. Thus, if our distance from the centre of the galaxy is 10,000 parsecs, then a theory which expresses the motion in the ξ and the ζ directions as a sum of two simple harmonic terms (*cf.* equations (32) and (33)) must break down for heights greater than 3500 parsecs above the galactic plane.

11. *Estimation of the Tilt of the Axes of the Velocity Ellipsoid in the Meridian Plane.*—From the remarks made towards the end of § 10 it follows that we can estimate ψ^* (*cf.* equation (76)) by evaluating the right-hand side of (76) on substituting for α, β and γ according to (81). We find that

$$\tan 2\psi^* \simeq -\frac{\varpi}{z}, \tag{102}$$

or by (84)

$$\tan 2\psi^* \simeq -\sqrt{8}, \tag{103}$$

or $\psi^* \simeq 55°$. This is the value of ψ^* stated at the end of § 9. The magnitude of this tilt of the principal axes of the velocity ellipsoid in the meridian plane is therefore quite large. It would be interesting to have an observational verification of this theoretical prediction. If our distance from the galactic centre is 10,000 parsecs, the value (103) refers to a height of 3500 parsecs above the galactic plane.

12. *General Remarks.*—The main conclusions reached can be described as follows :—

The generalization of Lindblad's dynamical theory of star-streaming enables us (1) to remove the difficulty regarding the ratio of the axes of the velocity ellipsoid, (2) to predict a tilt of the principal axes of the velocity ellipsoid in the meridian plane for motions above the galactic plane, and (3) to infer the instability of the motions according to the kinematical description (*cf.* § 2) valid for the nearby stars.

Now, as we have pointed out in § 1, the general theory of stellar systems with cylindrical symmetry predicts the equality of the Π- and the Z-axes of the velocity ellipsoid, which is contrary to observation. From this circumstance it has been generally inferred * that the assumption of a steady state

* For instance, Eddington remarks, "It seems to be clearly established by observation that radial star-streaming exists ; but Jeans's investigation shows the

must be abandoned. But if we consider the small motions of the individual stars with respect to the local standard of rest, then, as we have seen (§ 8), the condition for a steady state does not require the equality of the axes in the meridian plane. This arises because there is a second integral I_2 in addition to the "Lindblad integral" I_1 (*cf.* equation (56)). It should be pointed out in this connection that if we consider stellar systems with cylindrical symmetry, and if we further assume that the potential is due to a uniform ellipsoidal distribution of mass, then, as Clark * has shown, the equations of motion admit of one other integral in addition to the energy and the angular momentum integrals; Jeans's theorem does not now require the equality of the Π- and the Z-axes. Similarly, when we consider only small motions with respect to the local standard of rest, then to a first approximation there are two integrals I_1 and I_2. The existence of this second integral is somewhat analogous to the third energy integral in the z-co-ordinate which arises in the special case of uniform ellipsoidal distribution of mass. However, in the generalization of Lindblad's theory we need not postulate a special form for the potential function; instead we are required to consider only small motions with respect to the local centroid. It should be remembered in this connection that even for a value of z only slightly different from zero, though the solutions for ξ and ζ correspond to stability, yet the solution for η (equation (43)) includes a term which increases linearly with time. Hence, we do not strictly have a stable state of affairs for motions even slightly above the galactic plane. The "instability" due to the occurrence of the term Qt in the solution for η is of a "harmless" kind, since the motions $\dot{\xi}$, $\dot{\eta}$ and $\dot{\zeta}$ (*cf.* equations (40), (41) and (42)) are of a stable kind. On the other hand, when $\beta^2 > \alpha\gamma$ we have terms in the solutions which increase exponentially with time, and under these circumstances a state of motions quite different from that obtaining in the neighbourhood of the Sun must exist. Though instability, in the sense we have used this term, sets in only when $\beta^2 > \alpha\gamma$, we should expect that the method of approximation adopted for solving the dynamical problem must break down for heights substantially less † than what corresponds to $\beta^2 = \alpha\gamma$, due to the occurrence of the terms in $(\alpha\gamma - \beta^2)^{-1}$ in the solutions for ξ and ζ. Finally, the tilt of the axes in the meridian plane (predicted in the generalization of the Lindblad theory) is a feature not shared by stellar systems with cylindrical symmetry of the kind studied by Clark.

13. *Summary.*—The treatment of the observed stellar motions shows that the axis of preferential motion—the largest axis of the velocity ellipsoid—lies in the galactic plane and is directed towards the galactic centre (this is the radial axis). On the other hand, the theoretical study of stellar systems, with cylindrical symmetry and assumed to be in a steady state, has hitherto

price that must be paid—the galaxy cannot be in a steady state" (*Halley Lecture*, 1930).

* G. L. Clark, *M.N.*, **97**, 182, 1937.

† *I.e.* for heights less than 3500 parsecs above the galactic plane, assuming that our distance from galactic centre is 10,000 parsecs.

led to the result that the radial axis and the axis perpendicular to the galactic plane (the polar axis) are equal.

By a generalisation of Lindblad's theory of star-streaming the paper removes this inconsistency and demonstrates that, for a steady state of the system, the radial and polar axes are unequal. The paper also predicts for a group of stars at a considerable height above the galactic plane a definite tilt of the principal axes of the velocity ellipsoid in the meridian plane; in other words, the greatest axis is no longer parallel to the galactic plane.

Yerkes Observatory:
1938 *August* 24.

THE CORPUSCULAR ECLIPSE OF 1940 OCTOBER 1.

H.M. Nautical Almanac Office.

(Communicated by the Astronomer Royal)

In a previous paper (*M.N.*, **98**, 664–669, 1938) the paths of totality of the *visual* eclipse of 1940 October 1 were given for heights of 100 km. and 300 km., corresponding to the two ionized layers of the Earth's atmosphere, from which wireless waves are reflected. The present paper gives similar information about the *corpuscular* eclipse—that is to say, about the eclipse of corpuscles emitted from the Sun at a speed in the neighbourhood of 1000 miles a second. This is necessary to provide observers of the strength of wireless signals during the eclipse with complete information of the times of the phenomena, on each of the two theories as to the responsible ionizing agency.

The immediate effect of the assumption of the finite speed of 1000 miles a second for the corpuscles is to introduce large "aberration" terms into the positions of the Sun and Moon, which must naturally be uncorrected for normal aberration. These corrections are proportional to the product of the distance and the instantaneous motion in each co-ordinate and may be assumed constant for the duration of the eclipse. The values used in this eclipse are

$$\Delta\alpha_{\odot} = -232^{s}\cdot6 \qquad \Delta\delta_{\odot} = +1497'' \qquad \Delta\alpha_{\leftmoon} = -8^{s}\cdot9 \qquad \Delta\delta_{\leftmoon} = +43'',$$

corresponding to "light-times" of 1·0706 days for the Sun and 221 seconds for the Moon. It should be noted here that the net consequence of these corrections is to make the time of conjunction in right ascension about 2 hours earlier ($1^{h}\ 38^{m}$ for the eclipse under discussion), and that there is just the possibility that a visual eclipse will occur without a corresponding corpuscular eclipse and *vice versa*.

Light leaves the Sun from all points of its apparent disc, so that for the visual eclipse the umbral cone converges to a vertex whose position varies from just inside the Earth for a total eclipse to just outside it for an annular eclipse. Opinions appear to differ as to how corpuscles leave the Sun; on the assumption that they are all expelled radially, the Sun must be treated as a point-source, as was done by Miller * for the eclipse of 1932 August 31. It now appears clear that this assumption does not accord with observations, but no reliable data are available for the most suitable radiating diameter to adopt, though it is clear that this must be far less than that of the visual Sun and about the order of one-quarter of its apparent diameter. In both cases the resulting umbral cone either diverges or converges very much more slowly than for the visual eclipse,

* *M.N.*, **92**, 421, 1932.

with the result that the radius of the cone when it meets the Earth is comparable with that of the Moon instead of being extremely small. It is thus possible to adopt a constant radius for the shadow during the eclipse, as the variation in radius between the centre of the Earth and the subsolar point is negligible to the accuracy of the predictions required. The Sun can thus be exiled to an infinite distance, and the Moon replaced by a body of radius equal to the effective radius of the shadow cone at the centre of the Earth—or, more accurately, at a point between the centre and the subsolar point; apart from the motion of the Sun, the circumstances are identical with those for an occultation. The motion of the Sun is allowed for in the Besselian elements, which are naturally computed as for an eclipse and not for an occultation. This aspect of the problem immediately suggested the application of the occultation machine * to give rapid predictions for a large number of places on the Earth, and thus it has been possible to give far more detailed (but less accurate) predictions than in the case of a visual eclipse.

Predictions are required for both ionized layers, and it was also thought desirable to give some indication of the effect of the two possible assumptions concerning the apparent radiating surface of the Sun: in the first place the Sun was treated as a point-source and in the second as having an effective semi-diameter of 4′ (the Sun's apparent semi-diameter at the time of eclipse being 15′ 59″). Allowance for the varying height of the ionized layers is made simply by applying the appropriate scale factor either to the radius of the earth or inversely to the Besselian elements and to the effective radius of the Moon; the latter method is essential for application to the machine, as the size of the earth is fixed. The following table shows the four cases, together with the adjusted Besselian elements (in the form corresponding to prediction elements for occultations) for the time of conjunction in right ascension, and the corresponding effective radii of the Moon. The figures given are referred to the equatorial radius of the expanded Earth as unit, and the last column shows the actual radius of the shadow used on the machine, corresponding to an earth of 12 inches in diameter.

TABLE I

Case	Assumptions	Radius of Shadow	
		in Units	in Inches
1*a*	Point Sun, height 100 km.	0·2689	1·613
1*b*	Point Sun, height 300 km.	0·2609	1·565
2*a*	Sun's semi-diameter 4′, height 100 km.	0·2046	1·228
2*b*	Sun's semi-diameter 4′, height 300 km.	0·1985	1·191

Cases	T_0	H_0	d	Y	x'	y'
	h m	h m	° ′			
1*a*, 2*a*	11 13·72	23 27·91	−2 48·5	−0·3706	+0·5475	−0·1740
1*b*, 2*b*	11 13·72	23 27·91	−2 48·5	−0·3595	+0·5311	−0·1688

* Described and illustrated in *The Prediction and Reduction of Occultations*. Supplement to the *Nautical Almanac* for 1938.

The above figures are those used for the machine prediction, and they can be used as a basis for numerical prediction for any place not catered for in this paper.

Special stops corresponding to the radius of the shadow in inches were cut and used to stop down the lens on the occultation machine to the correct diameter. A rough drawing of the track of totality was made as the "circle" of light crossed the Earth, but no simple method could be devised to give a permanent record of the track. Since the eclipse track has been moved about 25° eastwards on the Earth's surface from the position for the visual eclipse, the corpuscular eclipse occurs in South America shortly after "sunrise", and it was not considered necessary to give detailed predictions for this portion of the track. A network of points, at intervals of 5° in latitude and longitude, was marked on the globe of the occultation machine, to cover the whole of the portion of South Africa traversed by the eclipse. Times of beginning and end of the eclipse at these points could then be read off, in exactly the same way as for the occultation of a star at a series of given stations. Independent numerical calculations were then made from the original Besselian elements for a number of these points, and the machine times corrected for the small systematic differences.

The accompanying tables (Tables II and III) give the corrected times of beginning and end of the corpuscular eclipse at all points in the network covering South Africa. Simple interpolation will enable the times for any particular place to be obtained to within about 2 or 3 minutes, though it must be stressed that the figures given are not necessarily correct to the nearest minute, owing mainly to the assumption of a spherical Earth. The times are, however, consistent among themselves, and the differences between the times of phenomena on different assumptions at the same place should be reliable.

The map (p. 732) shows the track of the corpuscular eclipse for case 1*b*—that is, for a point sun and a height of 300 km. The information given consists of the northern and southern limits of the band of totality, the "sunrise" and "sunset" curves and the outline of the shadow at hourly intervals. Although an adequate picture can be obtained by watching the path of the "circle" of light across the globe of the occultation machine, there is no reliable method of reading off the co-ordinates of the points through which it passes unless these have previously been plotted accurately. Consequently the curves shown have all been computed by the ordinary methods for eclipses, with the simplifications already introduced of a spherical Earth and a cylindrical shadow; the calculations have been checked by rough drawing on the globe and observation of the motion of the light on the machine. The map is intended to give a general impression, and so only the most important places have been marked on the outline map for identification purposes; the accuracy of the map and the drawing should enable times to be read off for any place to within about 10 minutes.

As can be seen from the relative figures in Tables II and III, the difference in times of beginning and end between heights of 100 km. and 300 km. is very small except near the end and edges of the track. The actual track

TABLE II

U.T. of Beginning of Total Corpuscular Eclipse

Long. / Lat.	E. 10°	E. 15°	E. 20°	E. 25°	E. 30°	E. 35°	E. 40°
	Case 1*a*—Point Sun, height 100 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 10	10 31	11 05					
15	10 26	10 44	11 08	11 38			
20	10 28	10 42	11 00	11 17	11 35	11 55	
25	10 34	10 45	10 59	11 13	11 27	11 40	11 53
30	10 42	10 52	11 03	11 13	11 24	11 35	11 45
35	10 53	11 00	11 09	11 17	11 26	11 34	
	Case 1*b*—Point Sun, height 300 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 10	10 28	10 58					
15	10 25	10 44	11 07	11 35			
20	10 28	10 43	11 00	11 18	11 37	11 56	
25	10 34	10 46	11 00	11 14	11 28	11 42	11 54
30	10 43	10 53	11 04	11 15	11 27	11 38	11 48
35	10 55	11 02	11 11	11 20	11 29	11 38	
	Case 2*a*—Sun of 8′ diameter, height 100 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 15	10 41	11 04					
20	10 40	10 57	11 15	11 35	12 02		
25	10 44	10 57	11 11	11 25	11 40	11 54	12 07
30	10 52	11 02	11 13	11 24	11 35	11 46	11 56
35	11 03	11 10	11 18	11 27	11 35	11 44	
	Case 2*b*—Sun of 8′ diameter, height 300 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 15	10 40	11 03					
20	10 40	10 56	11 15	11 36	12 02		
25	10 45	10 58	11 12	11 27	11 42	11 56	12 10
30	10 54	11 04	11 15	11 26	11 37	11 48	11 59
35	11 07	11 13	11 21	11 30	11 38	11 47	

at 100 km. would therefore hardly be distinguishable on the scale of the map from that given. For cases 2*a* and 2*b*, corresponding to an effective solar semi-diameter of 4′, no map has been given, but the general appearance of the umbral eclipse can easily be visualised by reducing the width of the track given in the ratio of 1 to 1·3, and retaining the other features more or less unchanged.

Finally, an attempt has been made to give some indication of the effect of a change in the assumed speed of 1000 miles a second for the corpuscles on the times of beginning and end. A speed of propagation of 1100 miles a second was assumed, Besselian elements computed and predictions made by the occultation machine for case 1*b*. The general effect is to displace the track given towards the south and the west; the times given in Tables II and III will be consistently later by about 14 minutes, the range of variation

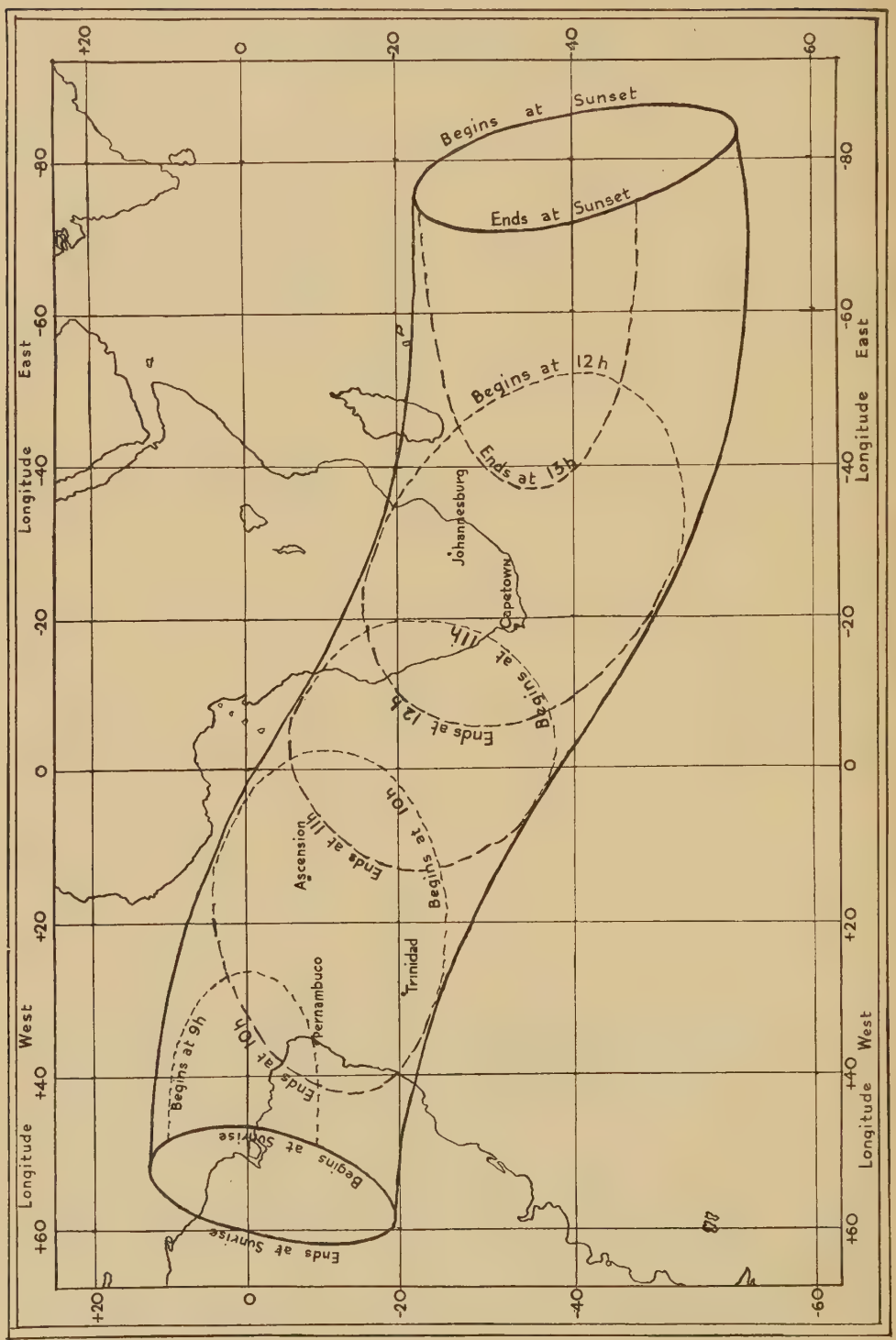
TABLE III

U.T. of End of Total Corpuscular Eclipse

Long. / Lat.	E. 10°	E. 15°	E. 20°	E. 25°	E. 30°	E. 35°	E. 40°
	Case 1*a*—Point Sun, height 100 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 10	11 22	11 18					
15	11 43	11 49	11 53	11 47			
20	11 56	12 04	12 13	12 19	12 23	12 25	
25	12 04	12 14	12 23	12 31	12 38	12 43	12 47
30	12 07	12 18	12 28	12 36	12 44	12 50	12 55
35	12 07	12 18	12 28	12 37	12 45	12 51	
	Case 1*b*—Point Sun, height 300 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 10	11 24	11 23					
15	11 45	11 52	11 56	11 54			
20	11 58	12 07	12 16	12 22	12 27	12 30	
25	12 06	12 17	12 26	12 34	12 41	12 47	12 51
30	12 09	12 21	12 31	12 40	12 47	12 53	12 58
35	12 08	12 20	12 31	12 40	12 48	12 54	
	Case 2*a*—Sun of 8′ diameter, height 100 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 15	11 28	11 31					
20	11 44	11 53	11 59	12 02	11 58		
25	11 53	12 04	12 12	12 20	12 26	12 30	12 32
30	11 57	12 08	12 18	12 26	12 34	12 40	12 45
35	11 56	12 07	12 18	12 28	12 36	12 42	
	Case 2*b*—Sun of 8′ diameter, height 300 km.						
°	h m	h m	h m	h m	h m	h m	h m
S. 15	11 30	11 33					
20	11 46	11 55	12 02	12 05	12 02		
25	11 55	12 06	12 15	12 23	12 29	12 34	12 37
30	11 58	12 10	12 21	12 30	12 37	12 43	12 49
35	11 56	12 09	12 20	12 30	12 38	12 45	

over the points given being between 10 and 16 minutes. For small variations in the assumed speed of propagation of the corpuscles, it may be taken that, over South Africa, the times of beginning and end of the eclipse, as given in the tables, should be modified by the addition of approximately $0^m.14 \times$ variation of speed in miles a second; this correction will operate for both positive and negative variations and for all four cases. The only Besselian elements in Table I which are affected by the change of speed of propagation are T_0, H_0 and Y; the new elements for case 1*b* corresponding to a speed of 1100 miles a second, and their variations for an increase of speed of one mile a second are:

$$T_0\ 11^h\ 22^m.67 \qquad H_0\ 23^h\ 36^m.86 \qquad \Delta T_0 = \Delta H_0 = +0^m.090$$
$$Y - 0.3503 \qquad \Delta Y = +0.000092.$$



CORPUSCULAR ECLIPSE OF 1940 OCTOBER 1

The variation ΔY must be multiplied by the small factor 1·03 for cases 1*a* and 2*a*, corresponding to a height of 100 km.

It is hoped that the information given will be sufficient for the general organization of observations, and perhaps for the explanation of the observed phenomena. All necessary material has been given for the calculation of approximate times for any particular station, but greater accuracy can be obtained by more elaborate methods.

This paper has been written as the result of opinions that predictions should be available for all possible assumptions as to the responsible ionizing agency. Acknowledgment is due to Professors Appleton and Chapman and Dr. Miller for their helpful advice in suggesting the various criteria to adopt. The work has been in the hands of Mr. W. E. Candler, who has either performed or supervised all the calculations.

Summary

A brief description is given of the adaptation of the occultation machine in the Nautical Almanac Office to the prediction of the corpuscular eclipse; times of beginning and end at a network of points covering South Africa are tabulated for the two heights of 100 km. and 300 km. and for two assumptions as to the effective radiating diameter of the Sun. A speed of propagation of 1000 miles a second is assumed for the particles, but an indication is given of the variation of the times for a change in the assumed speed.

ON THE MOTION OF THE APSIDAL LINE IN CLOSE BINARY SYSTEMS.

T. G. Cowling, M.A., D.Phil.

1. It was first pointed out in 1928 by H. N. Russell * that it is possible to obtain some information as to the density distribution in a star by observations of the advance of the apse-line in eclipsing binaries. Russell's formula was

$$\epsilon = k_1\left(\frac{R_1}{r}\right)^5\left(1+\frac{7m_2}{m_1}\right)+k_2\left(\frac{R_2}{r}\right)^5\left(1+\frac{7m_1}{m_2}\right), \qquad (1)$$

where ϵ is the ratio of the periods of orbital revolution and rotation of the apse-line, r the radius of the orbit, R_1 and R_2 are the radii of the two stars, m_1 and m_2 their masses, and k_1, k_2 are numerical constants depending on their density distributions. The value of the k's is 0·75 for a uniform star and 0 when the mass is all concentrated at the centre; for polytropic models of index n it is given by

$$k = \tfrac{1}{2}(\Delta_2(n) - 1), \qquad (2)$$

where $\Delta_2(n)$ is a function which has been tabulated by Chandrasekhar.†

Russell's formula was based on the assumption that the stars approximate to ellipsoids whose longest axes are along the line joining the centres of the two stars. K. Walter ‡ pointed out that if the stars are regarded as rigid bodies this assumption as to the direction of the axes is not valid for non-circular orbits, even if the orbital period coincides with the mean period of revolution of the stars: near periastron the radius vector joining the centres of the two stars must rotate faster than the stars, and *vice versa* at greater distances. This means that the longest axes of the ellipsoids oscillate on either side of the radius vector; Walter found that in consequence the apse-line must in general recede instead of advancing. Recently his work has been criticised by Z. Kopal,§ who claimed to have proved that Russell's formula is valid even when small librations of the axes are taken into account. His work appears, however, to rest on a misunderstanding; || there seems to be little doubt that Walter's result is correct if his assumption as to the rigidity of the stars is adopted.

That assumption is, however, extremely dubious. The time of adjust-

* *M.N.*, **88**, 642, 1928.

† *M.N.*, **93**, 449, 1933.

‡ *Veröff. der Universitäts-Sternwarte*, Königsberg, 3.

§ *M.N.*, **98**, 448, 1938.

|| Kopal considered librations of period large compared with the orbital period, which correspond roughly to the small oscillations of rigid ellipsoids, free to turn about their centres, under the action of their mutual gravitational attraction. He correctly showed that these do not, to the first order, affect the orbital motion;

ment of a star to an external gravitational field is of the order of the period of free adiabatic oscillations of the star, which is much smaller than the orbital period of another star round it. For this reason it would appear probable that the form of the star at any time closely approximates to the *equilibrium* form in which it is completely adjusted to the gravitational field of its companion ; in this case the longest axis of either ellipsoid is along the line joining the two centres, as Russell supposed. This is confirmed below (§§ 2–4) by detailed calculation in the special case of a uniform star. It does not follow that Russell's formula holds exactly, since Russell supposed that the two stars were invariable in form, while actually the tidal distortion of each varies with the varying distance between the stars. A different formula, taking this into account, is derived in § 5.

2. Consider two stars, centres O_1 and O_2, of masses m_1 and m_2, which consist of incompressible fluids of densities ρ_1 and ρ_2, and which are moving round each other in nearly circular paths. Let the stars rotate about axes O_1z_1 and O_2z_2 perpendicular to the plane of their orbits ; their angular velocities $\dot{\theta}_1$, $\dot{\theta}_2$ about these axes are defined as those which rigid masses of the same shapes and densities would need to be given in order to have the same moments of momentum about O_1z_1 and O_2z_2. Co-ordinate axes $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are taken, with O_1z_1 and O_2z_2 as axes, and rotating with angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ about these.

The distance r between the stars is assumed to be large compared with their dimensions ; this means that the stars can be regarded, with sufficient accuracy, as ellipsoids. These are supposed to result from the distortion of spheres of radii R_1 and R_2 when these are subjected to small uniform strains, such that, for example, the element dm_1 of m_1 moves from x_1, y_1, z_1 to $x_1+\xi_1$, $y_1+\eta_1$, $z_1+\zeta_1$, where *

$$\xi_1 = a_1x_1 + d_1y_1, \qquad \eta_1 = b_1y_1 + e_1x_1, \qquad \zeta_1 = c_1z_1. \tag{3}$$

The velocity of dm_1 relative to O_1 is made up of parts due respectively to the rotation of the axes $O_1x_1y_1z_1$ and to the displacement ξ_1, η_1, ζ_1 relative to these ; thus its components parallel to $O_1x_1y_1z_1$ are

$$\dot{a}_1x_1 + \dot{d}_1y_1 - (y_1+\eta_1)\dot{\theta}_1, \qquad \dot{b}_1y_1 + \dot{e}_1x_1 + (x_1+\xi_1)\dot{\theta}_1, \qquad \dot{c}_1z_1. \tag{4}$$

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but what Walter considered was librations identical in period with the orbital motion, and resulting therefrom : these Kopal did not discuss.

In passing, it may be observed that Kopal's discussion of the problem of constrained rotation (§ 3 of his paper) is open to objection. At the bottom of p. 451 he writes :

$$\int \frac{(1+e\cos\eta)^3}{e^2\sin^2\eta}d\eta = -3\eta + \text{periodic terms},$$

which is incorrect, since the expression on the left is always positive, and is infinite when the limits of integration are 0 and 2π. Actually Kopal's method of approximating to the orbital period by expanding in powers of K is inadmissible. A rather better method of approximating is indicated in § 5 below. Kopal's *result* in this section is perfectly correct, but his method is unsound.

* The validity of the assumption of a *uniform* strain can readily be established by detailed hydrodynamical arguments.

The total moment of momentum of m_1 about O_1z_1 is

$$\Sigma\{(x_1+\xi_1)(\dot{b}_1y_1+\dot{e}_1x_1+(x_1+\xi_1)\dot{\theta}_1)-(y_1+\eta_1)(\dot{a}_1x_1+\dot{d}_1y_1-(y_1+\eta_1)\dot{\theta}_1)\}dm_1,$$

and since this is, by the definition of $\dot{\theta}_1$, equal to

$$\dot{\theta}_1\Sigma\{(x_1+\xi_1)^2+(y_1+\eta_1)^2\}dm_1,$$

it follows that

$$\Sigma\{(x_1+\xi_1)(\dot{b}_1y_1+\dot{e}_1x_1)-(y_1+\eta_1)(\dot{a}_1x_1+\dot{d}_1y_1)\}dm_1=0;$$

or, since

$$\Sigma x_1^2dm_1=\Sigma y_1^2dm_1=\tfrac{1}{5}m_1R_1^2, \qquad \Sigma x_1y_1dm_1=0,$$
$$\dot{e}_1(1+a_1)+\dot{b}_1d_1-\dot{a}_1e_1-\dot{d}_1(1+b_1)=0.$$

Remembering that a_1, b_1, c_1, d_1 and e_1 are small, we can put

$$\dot{e}_1=\dot{d}_1, \qquad e_1=d_1. \tag{5}$$

Again, because the fluid is incompressible,

$$\frac{\partial\xi_1}{\partial x_1}+\frac{\partial\eta_1}{\partial y_1}+\frac{\partial\zeta_1}{\partial z_1}=0,$$

and so

$$a_1+b_1+c_1=0. \tag{6}$$

The originally spherical surface of m_1 has undergone a radial displacement whose magnitude at x_1, y_1, z_1 is

$$(x_1\xi_1+y_1\eta_1+z_1\zeta_1)/R_1=(a_1x_1^2+b_1y_1^2+c_1z_1^2+2d_1x_1y_1)/R_1. \tag{7}$$

Thus the gravitational potential of m_1 differs from that of a sphere of radius R_1 by the potential of a layer of surface density ρ_1 times the expression (7) over the surface of the sphere; the potential of m_1 at internal points accordingly is

$$\tfrac{1}{2}Gm_1(r_1^2-3R_1^2)/R_1^3-\tfrac{3}{5}Gm_1(a_1x_1^2+b_1y_1^2+c_1z_1^2+2d_1x_1y_1)/R_1^3,$$

where $r_1^2=x_1^2+y_1^2+z_1^2$. The potential energy of m_1 due to its own gravitational attraction is most simply found from this by calculating the work done in the displacement from the spherical form; it is

$$V_1=-\frac{Gm_1^2}{R_1}\{\tfrac{3}{5}-\tfrac{2}{25}(a_1^2+b_1^2+c_1^2+2d_1^2)\};$$

or, using (6),

$$V_1=-\frac{Gm_1^2}{R_1}\{\tfrac{3}{5}-\tfrac{1}{25}(3c_1^2+f_1^2+4d_1^2)\}, \tag{8}$$

where

$$f_1=a_1-b_1. \tag{9}$$

The potential energy due to the mutual attraction of m_1 and m_2 is similarly found; correct to terms of order $Gm_1m_2R_1^2/r^3$ and $Gm_1m_2R_2^2/r^3$, it is

$$V_{12}=-\frac{Gm_1m_2}{r}\Big\{1+\frac{3R_1^2}{5r^2}(a_1\cos^2(\theta-\theta_1)+b_1\sin^2(\theta-\theta_1)+d_1\sin 2(\theta-\theta_1))$$

$$+\frac{3R_2^2}{5r^2}(a_2\cos^2(\theta-\theta_2)+b_2\sin^2(\theta-\theta_2)+d_2\sin 2(\theta-\theta_2))\Big\}$$

$$=-\frac{Gm_1m_2}{r}\Big\{1+\frac{3R_1^2}{10r^2}(-c_1+f_1\cos 2(\theta-\theta_1)+2d_1\sin 2(\theta-\theta_1))$$

$$+\frac{3R_2^2}{10r^2}(-c_2+f_2\cos 2(\theta-\theta_2)+2d_2\sin 2(\theta-\theta_2))\Big\}, \qquad (10)$$

where θ_1, θ_2 and θ are the angles which O_1x_1, O_2x_2 and r make with a fixed direction in their plane. The terms involving R_1^2/r^2 in this arise from the moments and products of inertia of m_1 with respect to $O_1x_1y_1z_1$; since the terms vanish for an undisturbed sphere, they can be expressed in terms of the differences between the principal moments of inertia of m_1 at O_1. The total potential energy of the system is the sum of V_1, V_{12} and the gravitational potential energy V_2 of m_2.

The total kinetic energy can similarly be divided into parts T_1, T_2 and T_{12}, giving the energies of the motion of m_1 and m_2 relative to their mass-centres, and the energy of motion of the mass-centres. We have

$$T_{12}=\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(\dot r^2+r^2\dot\theta^2), \qquad (11)$$

$$\begin{aligned}T_1 &=\tfrac{1}{2}\Sigma\{(\dot a_1x_1+\dot d_1y_1-\dot\theta_1(y_1+\eta_1))^2+(\dot b_1y_1+\dot d_1x_1+\dot\theta_1(x_1+\xi_1))^2+\dot c_1^2z_1^2\}dm_1\\ &=\tfrac{1}{10}m_1R_1^2\{(\dot a_1-\dot\theta_1d_1)^2+(\dot d_1-\dot\theta_1-\dot\theta_1b_1)^2+(\dot b_1+\dot\theta_1d_1)^2\\ &\qquad\qquad+(\dot d_1+\dot\theta_1+\dot\theta_1a_1)^2+\dot c_1^2\}\\ &=\tfrac{1}{20}m_1R_1^2\{(2\dot d_1+f_1\dot\theta_1)^2+(\dot f_1-2d_1\dot\theta_1)^2+\dot\theta_1^2(2-c_1)^2+3\dot c_1^2\}, \qquad (12)\end{aligned}$$

with a similar expression for T_2.

3. From (8), (10), (11) and (12) we can write down the Lagrangian equations of motion. After dividing by constant factors, these take the forms

$$\ddot c_1+\tfrac{1}{3}(2-c_1)\dot\theta_1^2=-\frac{4}{5}\frac{Gm_1}{R_1^3}c_1-\frac{Gm_2}{r^3}, \qquad (13)$$

$$\ddot f_1-4\dot d_1\dot\theta_1-2d_1\ddot\theta_1-f_1\dot\theta_1^2=-\frac{4}{5}\frac{Gm_1}{R_1^3}f_1+\frac{3Gm_2}{r^3}\cos 2(\theta-\theta_1), \qquad (14)$$

$$\ddot d_1+\dot f_1\dot\theta_1+\tfrac{1}{2}f_1\ddot\theta_1-d_1\dot\theta_1^2=-\frac{4}{5}\frac{Gm_1}{R_1^3}d_1+\frac{3Gm_2}{2r^3}\sin 2(\theta-\theta_1), \qquad (15)$$

$$\frac{d}{dt}\{(2-c_1)^2\dot\theta_1+f_1(2\dot d_1+f_1\dot\theta_1)-2d_1(\dot f_1-2d_1\dot\theta_1)\}$$

$$=-\frac{6Gm_2}{r^3}(-f_1\sin 2(\theta-\theta_1)+2d_1\cos 2(\theta-\theta_1));$$

or, neglecting products like $f_1\dot d_1$, etc., in comparison with larger terms,

$$\frac{d}{dt}\{(1-c_1)\dot\theta_1\}=-\frac{3Gm_2}{2r^3}(-f_1\sin 2(\theta-\theta_1)+2d_1\cos 2(\theta-\theta_1)); \qquad (16)$$

$$\ddot{r} - r\dot{\theta}^2 = -\frac{G(m_1+m_2)}{r^2}\left\{1 + \frac{9R_1^2}{10r^2}(-c_1 + f_1\cos 2(\theta-\theta_1) + 2d_1 \sin 2(\theta-\theta_1)) + \frac{9R_2^2}{10r^2}(-c_2 + f_2\cos 2(\theta-\theta_2) + 2d_2 \sin 2(\theta-\theta_2))\right\}; \quad (17)$$

$$\frac{d}{dt}(r^2\dot{\theta}) = \frac{3G(m_1+m_2)}{5r^4}\{R_1^2(-f_1 \sin 2(\theta-\theta_1) + 2d_1\cos 2(\theta-\theta_1)) + R_2^2(-f_2 \sin 2(\theta-\theta_2) + 2d_2\cos 2(\theta-\theta_2))\}. \quad (18)$$

The "equilibrium" form of m_1 under the gravitational attraction of m_2 and the centrifugal force due to its own rotation can be deduced from (13), (14) and (15) by omitting the terms involving $\ddot{c}_1$, $\dot{f}_1$, $\ddot{d}_1$, $\ddot{\theta}_1$, $\dot{f}_1$ and $\dot{d}_1$. It is given by

$$c_1\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \tfrac{1}{3}\dot{\theta}_1^2\right) = -\frac{Gm_2}{r^3} - \tfrac{2}{3}\dot{\theta}_1^2, \quad (19)$$

$$f_1\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \dot{\theta}_1^2\right) = \frac{3Gm_2}{r^3}\cos 2(\theta-\theta_1), \quad (20)$$

$$d_1\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \dot{\theta}_1^2\right) = \frac{3Gm_2}{2r^3}\sin 2(\theta-\theta_1). \quad (21)$$

If $\dot{\theta}_1$ is of the same order as the orbital angular velocity $\dot{\theta}$, $\dot{\theta}_1^2$ is small compared with $4Gm_1/5R_1^3$ (see equation (22)): thus these equations approximate to

$$c_1 = -\frac{5m_2R_1^3}{4m_1r^3} - \frac{5R_1^3\dot{\theta}_1^2}{6Gm_1}, \quad (19')$$

$$f_1 = \frac{15m_2R_1^3}{4m_1r^3}\cos 2(\theta-\theta_1), \quad (20')$$

$$d_1 = \frac{15m_2R_1^3}{8m_1r^3}\sin 2(\theta-\theta_1). \quad (21')$$

The equilibrium form is realised in the state of relative rest, when $\theta_1 = \theta_2 = \theta$, and the stars move with forms unchanged in circular paths about the mass-centre of the system. In this case, by (17),

$$\dot{\theta}^2 = \frac{G(m_1+m_2)}{r^3}\left\{1 + \frac{9}{10r^2}[R_1^2(f_1-c_1) + R_2^2(f_2-c_2)]\right\}; \quad (22)$$

and so approximately

$$c_1 = -\frac{5R_1^3}{12r^3}\left(2 + 5\frac{m_2}{m_1}\right), \quad (23)$$

$$f_1 = \frac{15m_2R_1^3}{4m_1r^3}, \quad (24)$$

while $d_1 = 0$. From these we can calculate the semi-axes of the ellipsoid, which (*cf.* (3)) are now given by $R_1(1+a_1)$, $R_1(1+b_1)$, $R(1+c_1)$. The values obtained agree with the usual formulæ.

4. Next we consider small oscillations about the state of relative rest. To do so, we replace c_1, etc., in (13)–(18) by c_1+c_1', etc., where the new c_1 is the value in the state of relative rest and c_1' is the deviation from it in the oscillation; squares and products of c_1', etc., are ignored. Then we find that

$$\ddot{c}_1' + c_1'\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \frac{1}{3}\dot{\theta}^2\right) + \tfrac{2}{3}(2-c_1)\dot{\theta}\dot{\theta}_1' = \frac{3Gm_2 r'}{r^4}, \tag{25}$$

$$\ddot{f}_1' + f_1'\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \dot{\theta}^2\right) - 4\dot{d}_1'\dot{\theta} - 2f_1\dot{\theta}\dot{\theta}_1' = -\frac{9Gm_2 r'}{r^4}, \tag{26}$$

$$\ddot{d}_1' + d_1'\left(\frac{4}{5}\frac{Gm_1}{R_1^3} - \dot{\theta}^2\right) + \dot{f}_1'\dot{\theta} + \tfrac{1}{2}f_1\ddot{\theta}_1' = \frac{3Gm_2}{r^3}(\theta' - \theta_1'), \tag{27}$$

$$(1-c_1)\ddot{\theta}_1' - \dot{c}_1'\dot{\theta} = \frac{3Gm_2}{r^3}(f_1(\theta' - \theta_1') - d_1'), \tag{28}$$

$$\ddot{r}' - r'\dot{\theta}^2 - 2r\dot{\theta}\dot{\theta}' = -\frac{9}{10}\frac{G(m_1+m_2)}{r^4}\{R_1^2(f_1' - c_1') + R_2^2(f_2' - c_2')\}$$

$$+\frac{2G(m_1+m_2)r'}{r^3}\left\{1 + \frac{9}{5r^2}(R_1^2(f_1-c_1) + R_2^2(f_2-c_2))\right\}, \tag{29}$$

$$r\ddot{\theta}' + 2\dot{r}'\dot{\theta} = \frac{6}{5}\frac{G(m_1+m_2)}{r^5}\{R_1^2(d_1' - f_1(\theta' - \theta_1')) + R_2^2(d_2' - f_2(\theta' - \theta_2'))\}. \tag{30}$$

These, with four similar equations involving c_2', f_2', d_2' and θ_2', are the equations determining the small oscillations. To fix the periods $2\pi/\sigma$ of normal oscillations, we insert in them $\dot{c}_1' = \iota\sigma c_1'$, $\ddot{c}_1' = -\sigma^2 c_1'$, etc., and then eliminate the ten quantities c_1', c_2', f_1', f_2', d_1', d_2', θ_1', θ_2', r' and θ'; an equation of tenth degree in σ^2 results.

Three obvious solutions of (25)–(30) exist:

(i) With $\dot{\theta}' = \dot{\theta}_1' = \dot{\theta}_2' =$ constant, and the variables other than θ', θ_1' and θ_2' constants; this corresponds to a state of relative rest with slightly different angular velocity.

(ii) With $\dot{d}_1' = -f_1\dot{\theta}_1' =$ constant, and the variables other than d_1' and θ_1' constants; this corresponds to a state in which m_1 has an angular velocity different from $\dot{\theta}$, though its major axis remains pointing towards O_2.

(iii) With $\dot{d}_2' = -f_2\dot{\theta}_1' =$ constant, and the variables other than d_2' and θ_2' constants; this corresponds to a similar motion of m_2. For each of these three solutions we have $\sigma = 0$; thus only seven non-zero values of σ^2 can be found.

The oscillations corresponding to these non-zero values can be divided into three groups:

(i) For three oscillations σ^2 is nearly equal to $4Gm_1/5R_1^3$. These approximate to free oscillations of m_1 in the absence of m_2. Two are given by

$$\sigma^2 \doteq \frac{4Gm_1}{5R_1^3} \pm 2\dot{\theta}\left(\frac{4Gm_1}{5R_1^3}\right)^{\frac{1}{2}}, \qquad f_1' = \mp 2\iota d_1',$$

the variables other than f_1' and d_1' being small; the third by

$$\sigma^2 \doteq \frac{4Gm_1}{5R_1^3} - \tfrac{1}{3}\dot{\theta}^2,$$

all the variables save c_1' being small.

(ii) There are three similar oscillations which approximate to the free oscillations of m_2.

(iii) There is one oscillation for which σ^2 is nearly equal to $\dot{\theta}^2$. The variation of r' in this oscillation approximates to the variation of the radius vector when point-masses m_1 and m_2 are describing ellipses of small eccentricity under their mutual attraction. If we put

$$\sigma = \dot{\theta}(1-\epsilon), \tag{31}$$

where ϵ is small, then ϵ is the ratio of periods of orbital revolution and advance of the apse-line, as in (1).

It follows from (25)–(30) that for the last oscillation θ' and r'/r are of the same order of magnitude, while c_1', f_1', d_1' and θ_1' are of the order of $R_1^3 r'/r^4$. Thus in (25), (26) and (27) the terms involving $4Gm_1/5R_1^3$ are large compared with the other terms on the left, and so these equations give approximately

$$c_1' = \frac{15}{4}\frac{m_2 R_1^3 r'}{m_1 r^4}, \qquad f_1' = -\frac{45}{4}\frac{m_2 R_1^3 r'}{m_1 r^4}, \qquad d_1' = \frac{15}{4}\frac{m_2 R_1^3}{m_1 r^3}(\theta' - \theta_1'). \tag{32}$$

It may be observed that these are the values given by the "equilibrium" equations (19′)–(21′) when r, θ, θ_1 undergo small variations r', θ', θ_1', and so the form of m_1 approximates to the equilibrium form, in agreement with the general remarks in § 1. From (24) and (32) it follows that $d_1' = f_1(\theta' - \theta_1')$, and so, substituting in (30),

$$r\ddot{\theta}' + 2\dot{r}'\dot{\theta} = 0, \quad \text{or} \quad -\sigma^2 r\theta' + 2\iota\sigma r'\dot{\theta} = 0, \tag{30′}$$

the terms omitted in the approximation being of order less than $(R_1/r)^5$ or $(R_2/r)^5$ compared with those retained. Equation (30′) means that the law of areas remains correct to this order of approximation. Similarly, (29) becomes

$$r'\left\{\sigma^2 + \dot{\theta}^2 + \frac{2G(m_1+m_2)}{r^3}\left[1 + \frac{9}{5r^2}(R_1^2(f_1-c_1) + R_2^2(f_2-c_2))\right]\right.$$
$$\left. + \frac{27}{2}\frac{G(m_1+m_2)}{r^8}\left[\frac{m_2}{m_1}R_1^5 + \frac{m_1}{m_2}R_2^5\right]\right\} + 2\iota\sigma r\dot{\theta}\theta' = 0;$$

or approximately, using (22) and (30′),

$$r'\left\{\sigma^2 + \dot{\theta}^2 + 2\dot{\theta}^2\left[1 + \frac{9}{10r^2}(R_1^2(f_1-c_1) + R_2^2(f_2-c_2))\right]\right.$$
$$\left. + \frac{27}{2}\dot{\theta}^2\left[\frac{m_2}{m_1}\left(\frac{R_1}{r}\right)^5 + \frac{m_1}{m_2}\left(\frac{R_2}{r}\right)^5\right]\right\} - 4\dot{\theta}^2 r' = 0,$$

the approximation again being correct as far as terms of order $(R_1/r)^5$ and $(R_2/r)^5$. Using (28), this gives

$$\epsilon = \frac{9}{10r^2}(R_1^2(f_1 - c_1) + R_2^2(f_2 - c_2)) + \frac{27}{4}\left(\frac{m_2}{m_1}\left(\frac{R_1}{r}\right)^5 + \frac{m_1}{m_2}\left(\frac{R_2}{r}\right)^5\right);$$

or, substituting for f_1 and c_1 from (23) and (24),

$$\epsilon = \frac{3}{4}\left\{\left(1 + 16\frac{m_2}{m_1}\right)\left(\frac{R_1}{r}\right)^5 + \left(1 + 16\frac{m_1}{m_2}\right)\left(\frac{R_2}{r}\right)^5\right\}. \tag{33}$$

This differs from Russell's formula (1) by the factor 16 replacing his factor 7. The reason is simply that we have taken into account the variation of the tidal distortion with the mutual distance of the stars in the orbit.

5. Though it has been shown in only one case that the stars closely approximate to their equilibrium forms, there can be little doubt that this is true for any density distribution in the stars, since the proof depends simply on orders of magnitude. In the same way, the variations in the angular velocities $\dot\theta_1$ and $\dot\theta_2$ can be neglected in comparison with the variation of $\dot\theta$ in the general case. These facts enable us to generalise (33) to apply to other cases.

Substituting in (17) and (18) the equilibrium values of c_1, f_1 and d_1 given by (19′), (20′) and (21′), we find

$$\ddot r - r\dot\theta^2 = -\frac{G(m_1+m_2)}{r^2}\left\{1 + \frac{3R_1^5}{4r^2}\left(\frac{6m_2}{m_1r^3} + \frac{\dot\theta_1^2}{Gm_1}\right) + \frac{3R_2^5}{4r^2}\left(\frac{6m_1}{m_2r^3} + \frac{\dot\theta_2^2}{Gm_2}\right)\right\}, \tag{34}$$

$$\frac{d}{dt}(r^2\dot\theta) = 0, \tag{35}$$

where on the right-hand side of (34) we may regard $\dot\theta_1^2$ and $\dot\theta_2^2$ as constants. To generalise (34) to apply to non-uniform stars, we remember that the small terms involving c_1, f_1, d_1 in (10) can be expressed in terms of the differences between the principal moments of inertia of m_1, and for a non-uniform star these are $4k_1/3$ times as large as for a uniform star of the same mass and radius. Thus the generalised form of (34) for non-uniform stars is

$$\ddot r - r\dot\theta^2 = -\frac{G(m_1+m_2)}{r^2}\left\{1 + \frac{k_1R_1^5}{r^2}\left(\frac{6m_2}{m_1r^3} + \frac{\dot\theta_1^2}{Gm_1}\right) + \frac{k_2R_2^5}{r^2}\left(\frac{6m_1}{m_2r^3} + \frac{\dot\theta_2^2}{Gm_2}\right)\right\}$$

$$= -\frac{G(m_1+m_2)}{r^2}\{1 + \delta r^{-2} + \delta' r^{-5}\}, \text{ say.} \tag{36}$$

Equation (35) gives the usual integral

$$r^2\dot\theta = h. \tag{37}$$

Using this to eliminate the time from (36),

$$\frac{d^2u}{d\theta^2} + u = \frac{G(m_1+m_2)}{h^2}(1 + \delta u^2 + \delta' u^5),$$

where $u = 1/r$. This is satisfied if

$$u - \frac{G(m_1+m_2)}{h^2} = \left\{A - \frac{G(m_1+m_2)}{h^2}\int_0^\theta (\delta u^2 + \delta' u^5) \sin\theta d\theta\right\} \cos\theta + \left\{B + \frac{G(m_1+m_2)}{h^2}\int_0^\theta (\delta u^2 + \delta' u^5) \cos\theta d\theta\right\} \sin\theta. \quad (38)$$

A first approximation to the path is obtained by neglecting the terms involving δ and δ'. If the initial line of θ is suitably chosen, $B = 0$; then the first approximation is the ellipse

$$lu = 1 + e\cos\theta, \quad (39)$$

where $1/l = G(m_1+m_2)/h^2$, $e/l = A$. A second approximation is obtained on substituting from (39) into the right-hand side of (38). The resulting equation may be regarded as that of an ellipse with semi-latus rectum l, whose eccentricity and periastron longitude vary with θ. After one revolution, the periastron longitude has increased by

$$\tan^{-1}\left\{\int_0^{2\pi} (\delta u^2 + \delta' u^5) \cos\theta d\theta \Big/ \left[\frac{Ah^2}{G(m_1+m_2)} - \int_0^{2\pi} (\delta u^2 + \delta' u^5) \sin\theta d\theta\right]\right\};$$

or, using (39) and remembering that δ and δ' are small, approximately

$$\frac{G(m_1+m_2)}{Ah^2}\int_0^{2\pi} (\delta u^2 + \delta' u^5) \cos\theta d\theta = \frac{1}{e}\{\delta l^{-2} \cdot 2\pi e + \delta' l^{-5} \cdot 2\pi(\tfrac{5}{2}e + \tfrac{15}{4}e^3 + \tfrac{5}{16}e^5)\}.$$

Since this increase is the quantity earlier denoted by $2\pi\epsilon$, we find

$$\epsilon = \delta l^{-2} + \tfrac{5}{2}\delta' l^{-5}(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4);$$

or, substituting the values of δ and δ',

$$\epsilon = \frac{1}{l^2}\left(k_1 R_1^5 \frac{\dot\theta_1^2}{Gm_1} + k_2 R_2^5 \frac{\dot\theta_2^2}{Gm_2}\right) + \frac{15}{l^5}(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4)\left(k_1 R_1^5 \frac{m_2}{m_1} + k_2 R_2^5 \frac{m_1}{m_2}\right). \quad (40)$$

When the stars rotate with angular velocities equal to the mean orbital angular velocity,

$$\dot\theta_1^2 = \dot\theta_2^2 = G(m_1+m_2)(1-e^2)^3/l^3.$$

Thus in this case

$$\epsilon = k_1 \frac{R_1^5}{l^5}\left((1-e^2)^3\left(1 + \frac{m_2}{m_1}\right) + 15\frac{m_2}{m_1}(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4)\right) + k_2 \frac{R_2^5}{l^5}\left((1-e^2)^3\left(1 + \frac{m_1}{m_2}\right) + 15\frac{m_1}{m_2}(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4)\right). \quad (41)$$

When e is small, this reduces to

$$\epsilon = k_1\left(\frac{R_1}{r}\right)^5\left(1 + 16\frac{m_2}{m_1}\right) + k_2\left(\frac{R_2}{r}\right)^5\left(1 + 16\frac{m_1}{m_2}\right), \quad (42)$$

in agreement with (33).

6. Use of (41) or (42) instead of (1) reduces the values of k_1 and k_2

calculated from an observed value of ϵ by about a half. This corresponds to a much greater central condensation than is predicted by Russell's formula.

The only star for which an advance of periastron was, till recently, definitely observed was Y Cygni. For this, using Russell and Dugan's values * of R_1/r, R_2/r, m_2/m_1 and ϵ, and taking $k_1 = k_2$ in (42), we find $k_1 = 0{\cdot}016$, corresponding to a polytropic index only slightly less than 3 ; the ratio of central to mean density is 45, as compared with Russell and Dugan's value 16.

Recently Shapley and Miss Swope † have established the existence of apsidal rotation in GL Carinæ, for which they give

$$\epsilon = {\cdot}000263, \qquad R_1/r = R_2/r = 0{\cdot}215.$$

They do not give a value for m_1/m_2 ; if we assume $m_1 = m_2$ and $k_1 = k_2$, then we find from (42) the value $k_1 = 0{\cdot}0168$, corresponding to a ratio 42 of mean to central density ; if the masses are unequal, the ratio is increased. When allowance is made for the eccentricity of the orbit, the ratio is again increased ; but the effect is in this case slight.

Kopal ‡ has also considered other stars which possess a periodic property which may be due to rotation of the apse-line. For these, with the exception of S Equ, for which the results are doubtful, no star gives a ratio of central to mean density less than about 16 on the new formula, and the mean central condensation is not far from that of a polytrope of index 3. The calculated values of k_1 are admittedly open to wide errors, since the formula involves the fifth power of the quantities R_1/r and R_2/r, whose values are liable to an error which may be easily as much as 10 per cent. ; but we can definitely say that nothing in the observations of the motion of the apse-line indicates serious divergence from the density distribution predicted by Eddington.

One further remark may be made. Walter's analysis indicated the possibility of librations, of period large compared with the orbital period when the stars are relatively distant. By identifying periodic behaviour of the light-curves of β Lyræ and W Ursæ Majoris with the effect of these librations, he was able to obtain estimates of the density concentration. When the non-rigid nature of the stars is taken into account, however, it is seen that the gravitational field of the actual star affects its vibrations far more rapidly than the tidal field of the companion did in Walter's theory. The possible periods of librations are in consequence far smaller than Walter predicted : as we saw in § 4, the periods other than the orbital period are *small* compared with the orbital period. Walter's identification of the periodic behaviour of observed stars with the effect of librations is therefore invalid. We may therefore conclude that the only evidence of density distributions contradicting Eddington's theory which can be derived from binary stars is that derived from the ellipticity of the light-curves.§

* *M.N.*, **91**, 212, 1931.
† *Harvard Bulletin*, No. 909, p. 14, 1938.
‡ *Ast. Nach.*, **258**, 393, 1936.
§ This evidence, too, is perhaps not too reliable: see Luyten, *M.N.*, **98**, 459, 1938.

Summary.—It is pointed out that the discussions of the motion of the apse-line in binary stars given by Russell, Walter and Kopal all depend to a smaller or greater extent on an assumption that the stars can be regarded as rigid during the motion. When regard is paid to the fact that the stars are not rigid, a formula for the rate of advance of periastron is obtained, similar to Russell's, but differing in the numerical factor attached to certain of the terms. The density-concentrations of the stars which are deduced from the new formula are far more closely in agreement with Eddington's theory than those found by Russell. It is also noted that the librations of non-rigid stars are far more rapid than those of rigid stars: in consequence, the results regarding the density distribution derived by Walter from the periods of libration are not valid.

Manchester:
1938 *September* 20.

Erratum

M.N., **98,** Plate 6, *for* Scale 1 mm. = 12″·7 *read* Scale 1 mm. = 25″·4.

LIST OF ADDITIONS

TO THE

LIBRARY OF THE SOCIETY

1937 AUGUST 1 TO 1938 JULY 31

Serial Publications received in the regular course, and Reprints of Papers published in some of these Serials, are not included in this Annual List.

Unless otherwise indicated, the donor is the author or the Institution concerned.

An asterisk (*) indicates that the work is an excerpt.

A dagger (†) indicates that the work has been purchased.

Aarhus, Ole Rømer Observatoriet :
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(*C. T. Spencer.*) 8vo. London, 1937
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A simple form of periodogram. 8vo. *s.l.*, 1937

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†Amateur Telescope Making Advanced :
A sequel to Amateur Telescope Making. Albert G. Ingalls, Editor, Associate Editor, Scientific American. With contributions by numerous authorities.
8vo. Kingsport, U.S.A., 1937

Anděl (Karel) :
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See **Bok (B. J.).**

Bayer (Joannes) :
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Bulletin. A. Sciences mathématiques et physiques. Nos. 1, 2, 3.
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Botley (C. M.) :
The Air and its Mysteries. With a Foreword by Sir Richard Gregory.
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†The Observational Approach to Cosmology. 8vo. Oxford, 1937

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*On the wearing of the micrometer screw of the Wanschaff Zenith Telescope.
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(*E. Strömgren.*)

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See also **Brunn (A. von).**

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